

# Optimal Transfer to Solar – Terrestrial Collinear Libration Points

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Several missions are planned in Russia to launch spacecraft into the vicinity of the Solar-Terrestrial collinear libration points. The first of them is Spectr-Roentgen-Gamma intended to explore the sky in X-ray and Gamma-ray band. There are technical constraints on this project's realization, influencing the scenario of inserting the spacecraft into the operational orbit. One of these constraints is the location of the available ground station.

Due to the high latitude of Russian stations it is impossible to have visibility of the spacecraft from them during those phases of flight when the spacecraft is well below the ecliptic plane. To avoid this phenomenon, it is necessary to decrease the orbit amplitude in the direction orthogonal to the ecliptic plane. There are several methods how to do this including one with gravity assist maneuvers near Moon or use of the rocket engine for correction maneuvers, but the simplest – the search of an appropriate option within the family of possible transfer trajectories.

In the paper all these approaches are analyzed and it is shown that for standard scenario for them some difficulties do exist leading to the decrease the reliability of mission at large. The reason is that the thrust of the spacecraft rocket engines is too low, leading to burn durations that are too long.

For more practical approach, it is proposed to use the upper stage of the launcher for the maneuvers intended to decrease the amplitude of the orbit in the ecliptic pole direction. This leads to increasing the duration of spacecraft visibility to an acceptable level for the ground stations situated on the Russian territory. But the most promising and effective option is a single impulse trajectory with an optimal choice of the initial orbital state vector.

Also the problem of reaching the maximum amplitude of the orbit normal to the ecliptic, the “Z-amplitude”, is considered. This goal is to be reached for another project - Millimetron. This project goal is to build a space interferometer with very long base consisting of two telescopes: one in space near the [Sun-Earth L2] libration point and the other on the Earth's surface.

It is shown that it is possible to put the spacecraft into an orbit with a Z-amplitude value of more than one million kilometers, practically with the same velocity impulse as for orbits

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with several times less Z-amplitude. The proposed method for this is the appropriate choice of the perigee position of the transfer orbit.

### Nomenclature

$\mu_E$  – Earth gravitational constant,  $\mu_E=398600.435608 \text{ km}^3/\text{s}^2$ ;

$\mu_S$  – Sun gravitational constant  $\mu_S=132712.44002 \cdot 10^6 \text{ km}^3/\text{s}^2$ ;

$\mu_M$  – Moon gravitational constant  $\mu_M=4902.8 \text{ km}^3/\text{s}^2$ ;

$C_{20}$  – Earth flattening coefficient,  $C_{20}=1/298.257$ ;

$R_{Equat}$  – Earth equatorial radius,  $R_{Equat}=6378.137 \text{ km}$ ;

$\mathbf{r} = \mathbf{r}(t) = \{x, y, z\}$  – spacecraft radius vector in equatorial coordinate system ;

$\mathbf{v} = \mathbf{v}(t)$  – spacecraft velocity vector

$\mathbf{r}_S = \{x_S, y_S, z_S\}$  – Sun vector

$\mathbf{v}_s = \mathbf{v}_s(t)$  - Sun velocity vector

$\mathbf{r}_M = \{x_M, y_M, z_M\}$  – Moon vector

$\mathbf{v}_M = \mathbf{v}_M$  - Moon velocity vector

Point over variable means time derivative of the variable.

$\mathbf{X}(t) = (\mathbf{r}(t), \mathbf{v}(t))'$  – spacecraft state vector ( 3 coordinates of radius-vector and 3 of velocity vector) given as column)

$\Phi = \Phi(t, t_0)$  – 6x6 transfer matrix

### I. Introduction

**T**HIS paper was prepared as some description of the trajectory design for two projects now in development: SRG (Fig.1) and Millimetron.



**Figure 1. Spectr-Roentgen-Gamma spacecraft.**

The main goal of the SRG project is to map the sky in the X-ray and Gamma radiation bands.

For this purpose it is very important to minimize the noise level during measurements with the telescopes onboard the spacecraft in the chosen band. To reach the goal, initially high elliptical orbits were planned, analogous to the ones chosen for the INTEGRAL and Granat projects. But then it was accepted by the experimenters that the elliptical orbits are not optimal because the level of noise at the altitudes less than 60000 km is too high and there are bad influences even beyond this limit. An alternative low circular orbit was proposed under the Earth's radiation belts with height 580 km and an inclination as low as possible, as ideal – an equatorial orbit was considered. Low inclination allows avoiding the South Atlantic Magnetic Anomaly, where radiation belts reach very low heights above the Earth's surface. But to put a spacecraft into a nearly equatorial low orbit from Russian launching sites is practically impossible because the lowest latitude of available launching sites is higher than 48 degrees. With the use of Soyuz-2 -1B launch vehicle with Fregat upper stage and necessary spacecraft mass about 2200 kg the minimum achievable inclination was 28 degrees. This does not allow reaching a comfortable level of radiation and posed some difficulties on the appropriate platform development. Besides there would be a problem of control and receipt of telemetry that could not be solved by the ground stations on the territory of Russia.

For a solution of the problem, an orbit in the vicinity of the Solar-Terrestrial Collinear Libration Point L2 was chosen and a more powerful Zenith-2 launch vehicle with Fregat SB upper stage was proposed.

The Millimetron project was proposed as some more advanced version of the Radioastron project. The last one (now in flight) is intended for studies of the Universe with very high resolution with Very Long Base Interferometry. A long base is achieved by using spacecraft with a large (10 m diameter) parabolic antenna in a high elliptical orbit having a 350000-km apogee height. The minimum operational frequency of this radio telescope has a 5 cm wavelength. Millimetron is planned to be launched into an orbit in the vicinity of the L2 libration point and it will operate in millimeters wave band, thus its resolution is expected to be two orders of magnitude higher than for Radioastron.

## **II. Launch scenario and technical constraints**

It is known that to reach the orbit in the vicinity of the Solar-Terrestrial Libration points L1 or L2 from a low Earth orbit, only one velocity impulse is needed. If there are no constraints on the libration point orbit, then we can reach it by choosing only one parameter. For example the osculating semi major axis value at the moment of injection from the low Earth orbit may be used as such a parameter. All other parameters may be arbitrary under condition that they are inside some limits. In the real situation some technical constraints do exist and in our case we can not choose arbitrary the initial perigee altitude and inclination of the orbit. These parameters are determined by the initial low altitude parking orbit. The standard scenario of the launch uses putting the top block (upper stage plus spacecraft) onto this orbit with inclination equal to 51.5 degrees dictated mainly by the latitude of the launch site. The altitude of the parking orbit is determined by optimization of the payload of the launch vehicle that uses as low a height as possible allowed by the control system. To put the spacecraft into the transfer trajectory to the libration point, the upper stage is used. Usually to minimize the gravitational losses, more than one burn of the engine is planned, which means that one or two intermediate orbits are used in the procedure to reach the final transfer trajectory. Such an approach allows fulfilling correction maneuvers in perigee regions of the intermediate orbits, which decreases the total value of the correction maneuver  $\Delta V$ s that are needed. In any case for the design of the optimal transfer orbit, one has the following free osculating parameters: semi major axis value, right ascension of ascending node, argument of the perigee latitude. Practically to reach the chosen value of right ascension it is enough to choose the time of the launch, and the desired argument of perigee latitude is achieved by varying the time interval from launch to the start of the upper stage engine burn. And the needed value of the semi major axis is determined by the duration of the upper stage engine burn. In our studies we used a single impulse injection from the initial parking orbit onto the libration point transfer trajectory. Thus the task was to some extent simplified, but more precise calculations confirmed that the results obtained with the approximation described above are good enough for the purposes of orbit design studies.

## **III. Experimental demands and optimization criteria**

The planned experiments onboard the SRG spacecraft are to be controlled from ground stations on the territory of Russia and the key demands are to guarantee the possibility of contact with the spacecraft every day. Taking into account the volume of the telemetry information to be collected by spacecraft systems it is necessary to design an orbit that allows maximizing the duration of the total spacecraft visibility as seen from the ground stations. Because Russian ground stations are located in the North hemisphere and our orbit is in the vicinity of the L2 libration point the worst season for information receiving is the northern summer. And the worst case will be if the spacecraft will

reach the minimum ecliptic Z coordinate at the date of the summer solstice. It means that some phasing of the spacecraft motion along the Z axis is necessary, or the Z-amplitude of motion is to be minimized, or both.

The required phasing can be done by appropriate selection of the date of launch but such an approach puts constraints on the schedule of possible launches. So it is preferable to use other options to satisfy these requirements.

Contrary to the SRG project, for Millimetron a high Z-amplitude is required because in order to obtain good coverage of the explored objects, the area of coverage is to be filled by the trajectory projection on the plane as dense as possible and with maximum variation of the coordinates. On the other hand the requirement to keep in contact with spacecraft without use of Southern hemisphere ground stations is not applied here. Based on the same demand to complete the coverage, a large amplitude of motion along the X axis in the solar-ecliptic coordinate system is considered as an additional requirement.

#### IV. Tools and methods of studies

Since the launch of ISEE-3/ICE in 1978 [1] many other missions in vicinity of solar-terrestrial collinear libration points have been fulfilled and appropriate methods to design such missions have been developed. Most of these methods are based on use of analytical and semi analytical approaches of solving the restricted three body problem. In our studies we used the methods proposed in [2, 3] in order to receive some preliminary estimations and qualitative understanding of the tasks. But the design of the missions was done with the use of numerical methods which included the numerical integration of the system of differential equations describing the motion of the spacecraft, Sun and Moon in the Earth-centered non-rotating (inertial) equatorial coordinate system. The gravity field of the Earth was described with an accuracy up to the second term in the Legendre series.

This system of equations is given below and includes also the differential equations of isochronous variables which are used for the tracking accuracy estimations and necessary correction maneuver calculations.

In the expressions given below all vectors are presented by half bold letters. They are considered to be columns if they are not marked with a " ' " index. In this last case they are considered to be rows. Derivatives of scalar by vectors are considered to be a row vector. Transposition operation is marked by the " ' " index.

Differential equations of spacecraft motion for our problem may be written as:

$$\dot{\mathbf{r}} = \mathbf{v}$$

$$\dot{\mathbf{v}} = -\mu_E \frac{\mathbf{r}}{r^3} - \mu_S \frac{\mathbf{r} - \mathbf{r}_S}{|\mathbf{r} - \mathbf{r}_S|^3} - \mu_M \frac{\mathbf{r} - \mathbf{r}_M}{|\mathbf{r} - \mathbf{r}_M|^3} - \mu_S \frac{\mathbf{r}_S}{r_S^3} - \mu_M \frac{\mathbf{r}_M}{r_M^3} + \mathbf{F}_{Eobl},$$

$$\mathbf{F}_{Eobl} = -\frac{1.5}{r^5} \cdot C_{20} \cdot R_{Equat}^3 \cdot \left( 5 \frac{z^2}{r^2} - 1 \right) \cdot \mathbf{r} - \frac{3z}{r^5} C_{20} R_{Equat}^3 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

where the last vector is the acceleration vector due to the Earth's oblateness.

For the Moon the differential equations are:

$$\dot{\mathbf{r}}_M = \mathbf{v}_M,$$

$$\dot{\mathbf{v}}_M = -(\mu_E + \mu_M) \frac{\mathbf{r}_M}{r_M^3} - \mu_S \frac{\mathbf{r}_M - \mathbf{r}_S}{|\mathbf{r}_M - \mathbf{r}_S|^3} - \mu_S \frac{\mathbf{r}_S}{r_S^3} + \mathbf{F}_{Mobl},$$

where

$$\mathbf{F}_{Mobl} = -\frac{26.33234305}{r_M^5} \cdot \left( 5 \frac{z_M^2}{r_M^2} - 1 \right) \cdot \mathbf{r}_M - \frac{52.6646851 z_M}{r_M^5} \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

The Sun's motion is described by the equations:

$$\dot{\mathbf{r}}_S = \mathbf{v}_S,$$

$$\dot{\mathbf{v}}_S = -(\mu_E + \mu_S) \frac{\mathbf{r}_S}{r_S^3} - \mu_M \frac{\mathbf{r}_S - \mathbf{r}_M}{|\mathbf{r}_S - \mathbf{r}_M|^3} - \mu_M \frac{\mathbf{r}_M}{r_M^3}.$$

The transfer matrix  $\Phi$  gives in linear approximation the function of state vector at instant  $t$  from the state vector at instant  $t_0$ :

$$\Phi = \Phi(t, t_0) = \frac{\partial \mathbf{X}(t)}{\partial \mathbf{X}(t_0)} = \begin{bmatrix} \frac{\partial \mathbf{r}(t)}{\partial \mathbf{r}(t_0)} & \frac{\partial \mathbf{r}(t)}{\partial \mathbf{v}(t_0)} \\ \frac{\partial \mathbf{v}(t)}{\partial \mathbf{r}(t_0)} & \frac{\partial \mathbf{v}(t)}{\partial \mathbf{v}(t_0)} \end{bmatrix} = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix}.$$

This matrix is the solution of the differential matrix equation

$$\dot{\Phi} = \begin{bmatrix} \mathbf{0}_{3 \times 3} & \mathbf{I}_3 \\ \mathbf{F}_r & \mathbf{0}_{3 \times 3} \end{bmatrix} \Phi,$$

with initial condition

$$\Phi(t_0, t_0) = \mathbf{I}_6,$$

where

$$\begin{aligned} \mathbf{F}_r = \frac{\partial \dot{\mathbf{v}}}{\partial \mathbf{r}} = & -\mu_E \cdot \frac{1}{r^3} \left( \mathbf{I} - \frac{3}{r^2} \cdot \mathbf{r} \cdot \mathbf{r}' \right) - \\ & - \mu_S \cdot \frac{1}{|\mathbf{r} - \mathbf{r}_S|^3} \left( \mathbf{I} - \frac{3}{|\mathbf{r} - \mathbf{r}_S|^2} \cdot (\mathbf{r} - \mathbf{r}_S) \cdot (\mathbf{r} - \mathbf{r}_S) \right) - \\ & - \mu_M \cdot \frac{1}{|\mathbf{r} - \mathbf{r}_M|^3} \left( \mathbf{I} - \frac{3}{|\mathbf{r} - \mathbf{r}_M|^2} \cdot (\mathbf{r} - \mathbf{r}_M) \cdot (\mathbf{r} - \mathbf{r}_M) \right) - \\ & - 1.5 \cdot C_{20} \cdot R_{Equat}^3 \cdot \left[ \frac{10z}{r^7} \cdot (\mathbf{0} \ \mathbf{0} \ \mathbf{r}) + \frac{5}{r^7} \cdot \left( 1 - \frac{7z^2}{r^2} \right) \cdot \mathbf{r} \cdot \mathbf{r}' + \frac{1}{r^5} \left( \frac{5z^2}{r^2} - 1 \right) \cdot \mathbf{I} \right] - \\ & - 3 \cdot C_{20} \cdot R_{Equat}^3 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \cdot \left\{ \frac{1}{r^5} \cdot (0 \ 0 \ 1) - \frac{5z}{r^7} \cdot \mathbf{r}' \right\}. \end{aligned}$$

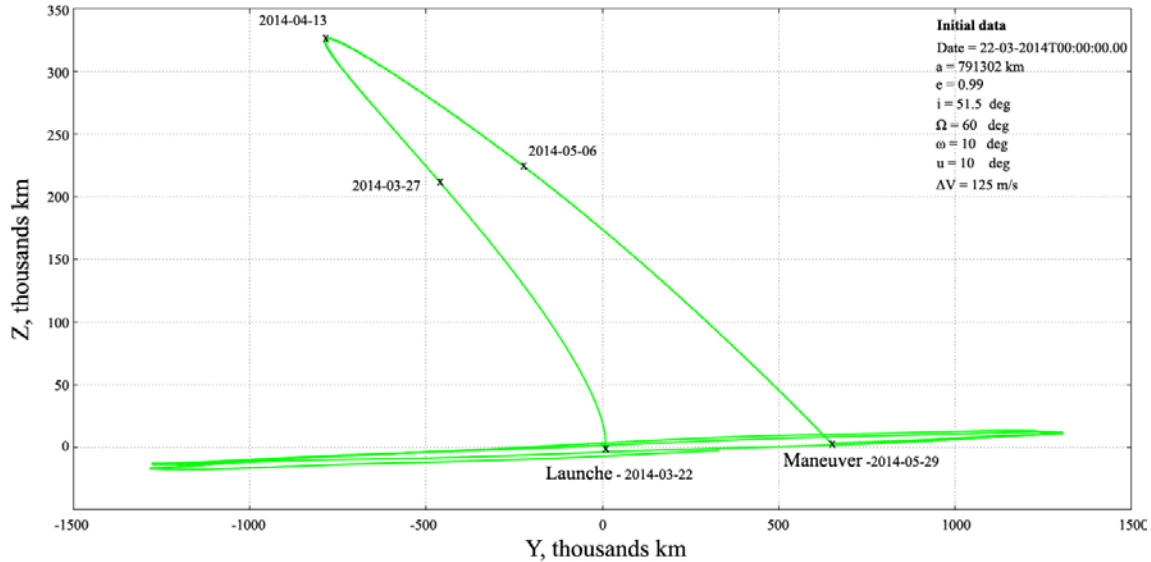
Solutions of the equations given above were found by numerical integration using the Runge-Kutta method with the initial values of the variables chosen on the basis of some preliminary heuristic estimations.

## V. Results of the studies

By use of the software tools described above, the family of the L2 libration point trajectories have been constructed for launches in each month of 2014. The initial osculating parameters of these trajectories have been chosen in such a way that allowed satisfying the main requirement: to achieve visibility each day of the mission from the Ussuriysk ground station and to maximize the total duration of visibility during the first two years of the mission.

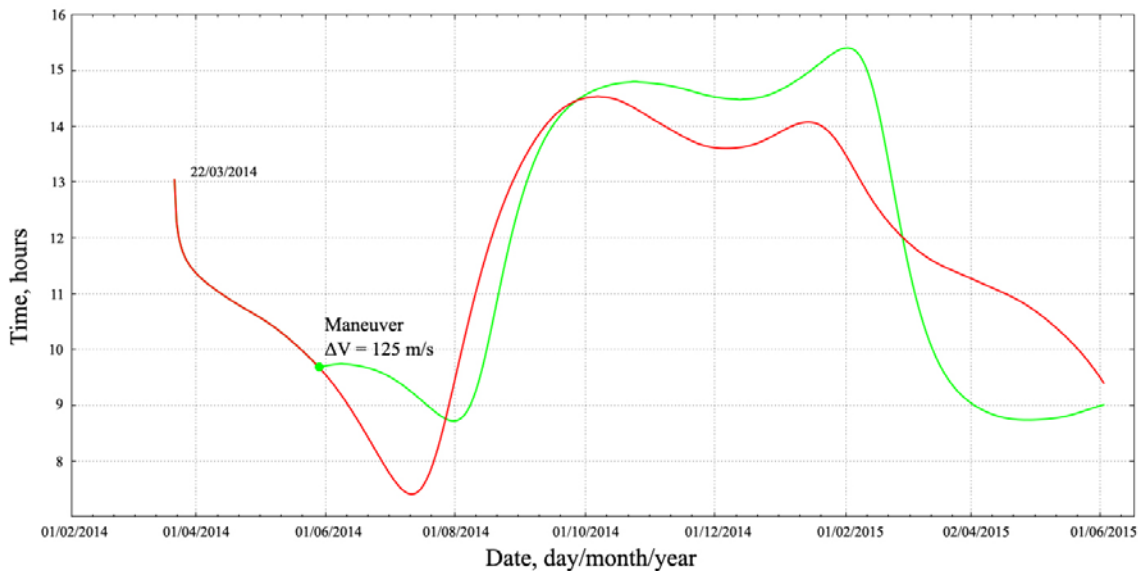
It is obvious that, if the libration point trajectory is very close to ecliptic plane, then we can avoid the most unfavorable situation when the spacecraft passes too low above the horizon at the ground station. Fig.2 illustrates what can be achieved if an orbital maneuver is performed, placing the spacecraft practically in the ecliptic plane by applying a velocity impulse almost equal to the negative of the Z component of velocity when the spacecraft is in

the ecliptic plane. For this and the following figures, the rotating solar-ecliptic coordinate system is used with centre at the Earth and the X axis directed from Earth to Sun, with the Z axis orthogonal to the ecliptic plane.



**Figure 2. Transfer trajectory with rocket maneuver.** This illustrates what can be achieved if an orbital maneuver is performed that puts the spacecraft practically in the ecliptic plane by applying a velocity impulse almost equal to the negative of the Z component of velocity when the spacecraft is in the ecliptic plane

Fig.3 gives the duration of visibility of the spacecraft from Ussuriysk as a function of dates for two cases: without maneuver and with applying the Z velocity impulse. One can see analyzing Fig. 3 that we practically do not receive any gains in total visibility after the maneuver because some initial rise of visibility duration is followed by quite visible losses later.



**Figure 3. Visibility from Ussuriysk station.** Figure shows two options: red – without maneuver, green – with maneuver.

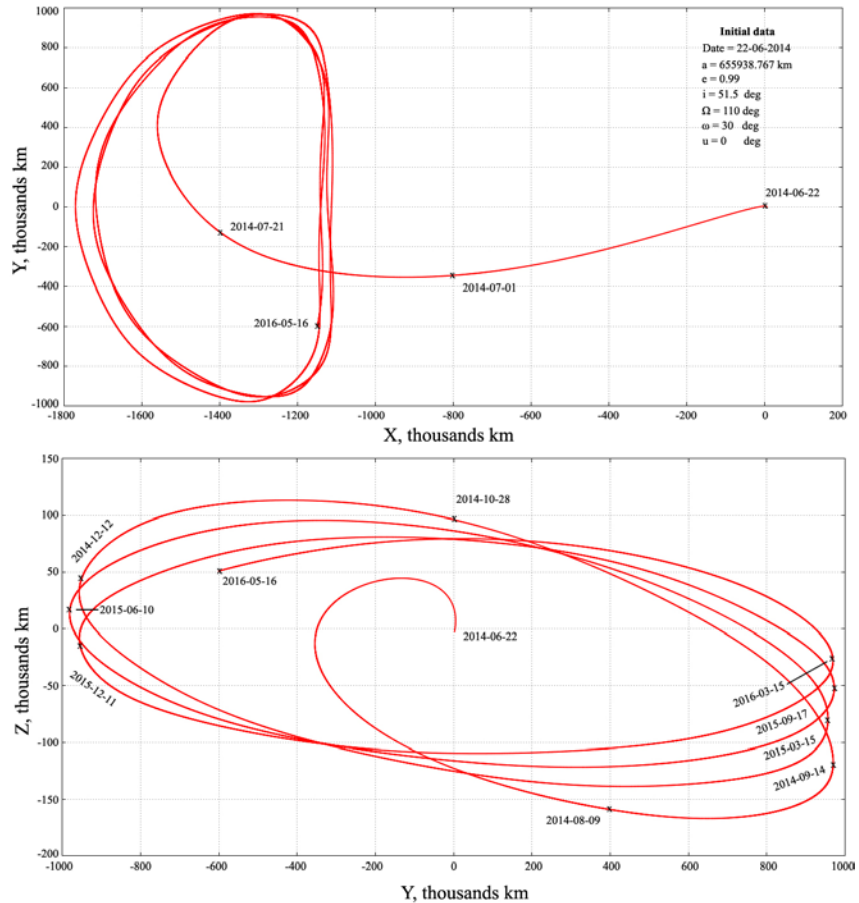
A series of calculations for different launch dates, optimizing the total duration of visibility from the Ussuriysk ground station have confirmed that with a Z amplitude not exceeding 250000 km the visibility duration is quite acceptable. Confirmation of reaching this Z amplitude is confirmed in Table 1. An example of such an orbit is given in Fig.4 (XY and YZ projections). This trajectory presents an acceptable combination of the phasing of the orbit

22-01-2014	22-02-2014	22-03-2014	22-04-2014	22-05-2014	22-06-2014
Z = -170 ths. km a=778675 km e=0.99 i=51.5 deg $\Omega=0$ deg $\omega=10$ deg u=0 deg	Z = -200 ths. km a=769478 km e=0.99 i=51.5 deg $\Omega=30$ deg $\omega=10$ deg u=0 deg	Z = -180 ths. km a=667669 km e=0.99 i=51.5 deg $\Omega=30$ deg $\omega=10$ deg u=0 deg	z = -200 ths. km a=643605 km e=0.99 i=51.5 $\Omega=45$ $\omega=10$ u=0	Z=-150 ths. km a=736775 km e=0.99 i=51.5 deg $\Omega=100$ deg $\omega=30$ deg u=0 deg	Z=-150 ths. km a=655938 km e=0.99 i=51.5 deg $\Omega=110$ deg $\omega=30$ deg u=0 deg
22-07-2014	22-08-2014	22-09-2014	22-10-2014	22-11-2014	22-12-2014
Z=-150 ths. km a=646815.5 km e=0.99 i=51.5 deg $\Omega=140$ deg $\omega=15$ deg u=0 deg	Z=-150 ths. km a=651499.7 km e=0.99 i=51.5 deg $\Omega=130$ deg $\omega=20$ deg u=0 deg	Z=-150 ths. km a=646982 km e=0.99 i=51.5 deg $\Omega=220$ deg $\omega=-15$ deg u=0 deg	Z=-150 ths. km a=643606.8 km e=0.99 i=51.5 deg $\Omega=210$ deg $\omega=-10$ deg u=0 deg	Z=-250 ths. km a=872162 km e=0.99 i=51.5 deg $\Omega=210$ deg $\omega=-10$ deg u=0 deg	Z=-200 ths. km a=692715 km e=0.99 i=51.5 deg $\Omega=330$ deg $\omega=-10$ deg u=0 deg

**Table 1. Initial osculation elements of the transfer trajectory.** The table confirms the possibility of selecting initial transfer orbit elements that allow achieving the needed visibility from Ussuriysk .

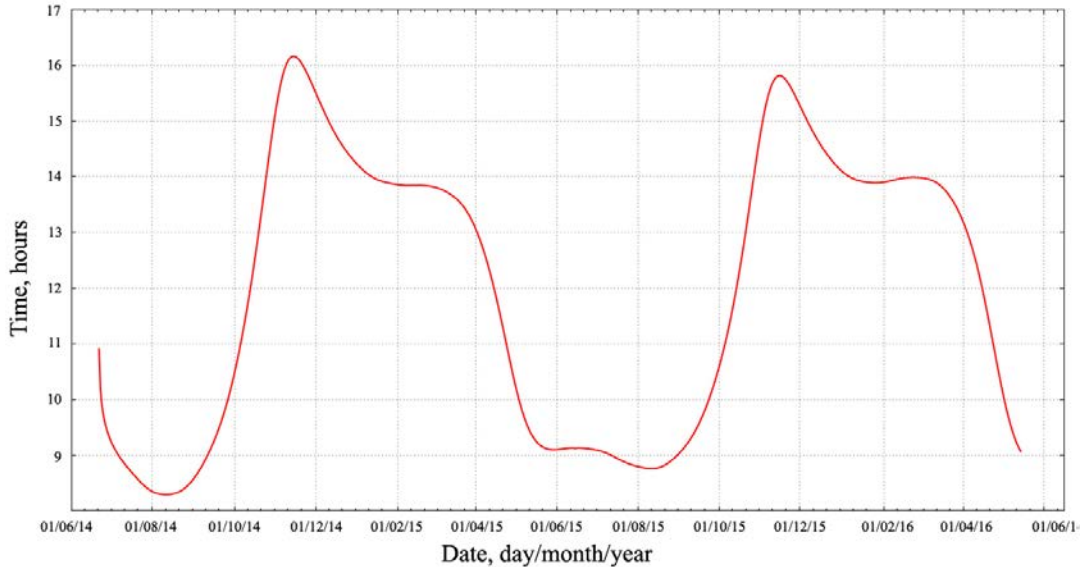
with the seasons, i.e. the minimum Z coordinate is achieved at the time far enough from the summer solstice. It is worth mentioning that optimal phasing of the spacecraft's position on the orbit is achieved if the maximum ecliptic Z coordinate is reached on the day of the summer solstice. The period of spacecraft motion in the L2 orbit is about half a year, so after three months, the spacecraft will be in the point of orbit with minimum Z but it will be not so critical because at that moment it will be at the spring equinox. Three months later the spacecraft will reach again the maximum Z but it will happen in the winter solstice, so we will receive the maximum visibility during the year. Thus the phase shift of the presented example from the optimum case is about 1.5 month. Fig.5 gives visibility duration as a function of dates for this orbit.

For future projects there are plans to launch small spacecraft to the vicinity of solar-terrestrial collinear libration points by a piggy-back mode. Such options were analyzed in [2]. For such cases we do not have the same freedom of choosing initial parameters of the orbit as for dedicated launches. For example the argument of perigee latitude may be dictated by the main payload demands.



**Figure 4. Example of trajectory to and near L2 point satisfying visibility requirements.**

In this case in order to reach the necessary L2 orbit parameters some additional maneuver may be needed. If as in previous cases, there are constraints on the Z coordinate, a maneuver is needed to satisfy them. One of the



**Figure 5. Visibility duration for the orbit presented in figure 4.**

approaches described for example in [3] is to perform such a maneuver after reaching the vicinity of the libration point. But the size of the velocity impulse during this maneuver may be restricted by the spacecraft thruster performance. So it is preferable to perform the maneuver with the launch vehicle upper stage.

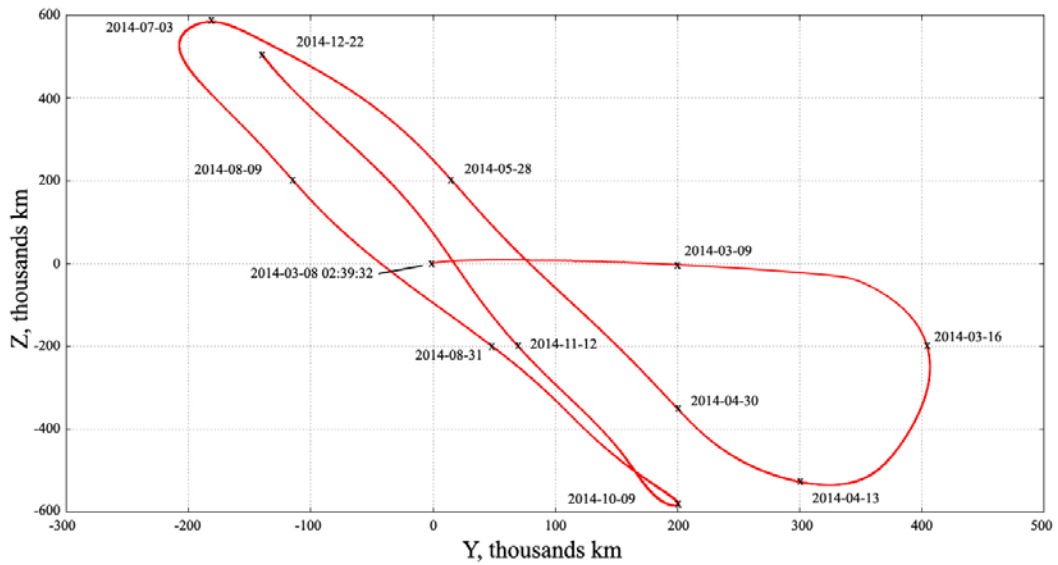
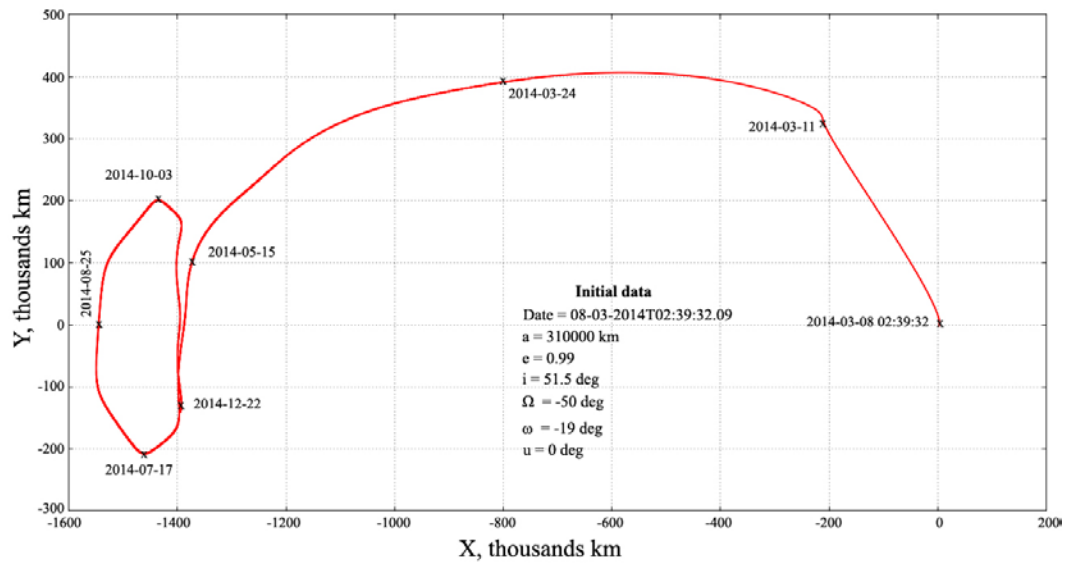
Studies have shown that it is possible and in addition this may give some gains in propellant consumption. Fig.5 illustrates the case when we reach L2 vicinity without maneuver, and then the Z coordinate is as low as minus 450000 km. With a maneuver impulse of only 100 m/s along Z this coordinate became only -100000 km if it is done 3 days after launch from low Earth orbit as is shown in Fig.6. Analysis has shown that for later maneuvers the value of the necessary delta-V increases (to 150 m/s for applying the impulse at 5.3 days after launch).

Another rather effective tool to reach the required parameters of the L2 orbit is the use of gravity assist maneuvers near the Moon as described in [4]. It allows decreasing the amplitude of the orbit to limits down to 200000 km along the Y axis as may be required by the spacecraft experiments. In our case the possibilities were studied to decrease this amplitude by such maneuver with a simultaneous increase of the Z amplitude. Fig. 6 confirms that it is doable; with a proper choice of initial trajectory parameters one can achieve a Z amplitude up to 600000 km while keeping the Y amplitude inside 200000km limits.

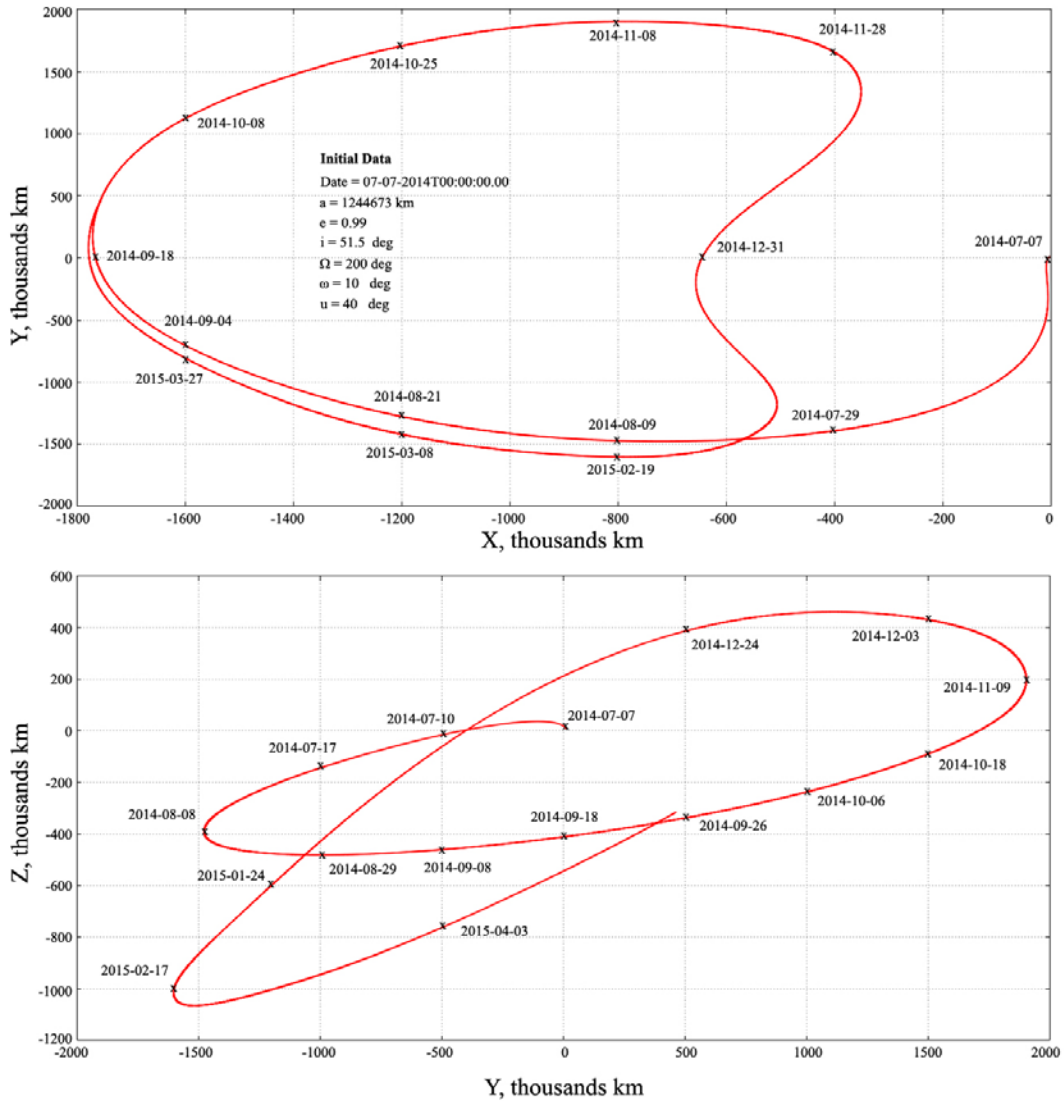
For the Millimetron project as described earlier the task is in some sense opposite to the ones analyzed above: how to reach a maximum amplitude in the direction orthogonal to the ecliptic plane. Like in the previous task the goal may be reached just by choice of the initial parameters of the transfer to libration point orbit.

In terms of initial osculating orbital elements with a high enough angle between the line of apsides and the ecliptic plane it is possible to achieve the orbit in the vicinity of the collinear libration point with a Z amplitude exceeding one million km. It is confirmed by the trajectory chosen and presented in Fig.7. The Z amplitude of this trajectory reaches 1100000 km. In addition one should take into account that this trajectory has also rather high amplitude along the X axis that is very beneficial for VLBI experiments.





**Figure 6. Trajectory with a gravity assist maneuver near the Moon.** Example of using a Lunar swing by to reach an L2 orbit with a high amplitude normal to ecliptic and small amplitudes in the ecliptic plane.



**Figure 7. High amplitude L2 orbit for VLBI Millimetron project.** *Illustration of a possibility to put a spacecraft into an L2 orbit with very high amplitudes along the Z and X axes.*

## VI. Conclusions

It has been shown that by an appropriate choice of the free initial parameters of the transfer trajectory, it is possible to reach orbits in the vicinity of the L2 solar-terrestrial libration point that allow a maximum duration of visibility from a Russian high-latitude ground station each day of the mission. No additional maneuvers for a nominal transfer trajectory are necessary during the spacecraft's flight to the libration point orbit. Only correction maneuvers are needed. Even for the most unfavorable season near the summer solstice point, the duration of visibility from the Ussuriysk ground station exceeds 7 hours per day.

If there are some constraints on the perigee argument of the transfer orbit that do not allow achieving a low enough L2 orbit amplitude, then an appropriate maneuver must be performed to decrease this amplitude to the required limit. It was shown that such a maneuver should be executed optimally in the vicinity of where the orbit crosses the ecliptic plane, preferably 3-5 days after launch. It was confirmed that such an approach is more beneficial in term of delta-V than to do the maneuver after reaching the vicinity of the L2 point.

For the experiments planned to be performed from an L2 orbit with a high amplitude along the direction normal to ecliptic, it is proposed to use a gravity assist maneuver near Moon, which allows reaching a Z amplitude value of 600000 km along with a rather small amplitude (200000 km) along the Y axis.

For the VLBI experiments using a space radio telescope as a component, there are requirements to maximize the amplitudes of the libration point orbit normal to the ecliptic and along the direction to the Sun. Our studies confirmed that it is doable using a single impulse transfer trajectory. With appropriate values of the initial argument of perigee, one can achieve an amplitude greater than 1100000 km along the normal to the ecliptic and 550000 km along the direction to the Sun.

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