Taking Advantage of Uncertainty in Mission Planning

Daniel X. Junker[1]
LSE Space GmbH, Aregelsrieder Feld 22, 82234 Wessling, Germany

Typical mission maneuver planning is taken up with designing the nominal burn sequence. For example, in a normal Geosynchronous Transfer Orbit (GTO), one is concerned with the effect on the trajectory from each burn, assuming all the previous have performed nominally. Principal effort is devoted to evaluating whether the resulting trajectory meets visibility, radio frequency interference requirements, solar constraints and so forth. Customarily one assumes that prevailing orbit determination uncertainty, engine performance and attitude control uncertainties are sufficiently small that problems encountered can be handled as they arise in flight. This assumption is exacerbated by the underlying belief that coherently handling all the sources of uncertainty requires too much computing time, too many runs of too many alternate cases to make the effort worthwhile. This need not be the case. It is precisely because the prevailing uncertainties are reasonably small that differential techniques can be applied to address the ensemble of prevailing uncertainties in the same computation used to propagate the planned nominal trajectory. One can readily determine, for example, what range of post burn longitude drift rates to expect. Equally quickly one can decide how to adjust nominal targets to comfortably handle the expected range of orbit, attitude and burn uncertainty. This paper discusses the technique and applies it to trajectories seen in recent missions. It is shown how the result is a more robust burn plan at remarkably low cost in the development process.

Nomenclature

a,e,i,Ω,ω	=	Keplerian Elements
E, M, f	=	Keplerian Eccentric Anomaly, Mean Anomaly and True Anomaly
μ	=	Gravitational constant
$\mathbf{R},\mathbf{V}$	=	Cartesian position and velocity vectors
r_x,r_y,r_z	=	Elements of the Cartesian position
v_x,v_y,v_z	=	Elements of the Cartesian velocity
r	=	Orbital radius, the magnitude of the vector $\mathbf{R}$
$\Delta v_r,\Delta v_w$	=	Delta-V delivered along the orbit radial and orbit normal
Δv_s	=	Delta-V delivered along the cross product of the orbit normal and the orbit radial

I. Introduction

THE main concern in maneuver planning is "what will happen if everything goes well?" The principal sources of uncertainty governing a plan are: engine performance, attitude control, and pre-maneuver trajectory uncertainty. Differential techniques can directly incorporate these into the main mission planning flow. That is, if you already have the means to propagate the orbit and make an ephemeris, you are just a few more computer subroutines away from computing the error bars on your plan. The error bars will help you quickly access whether or not your burn plan is robust.

[1] Senior Flight Dynamics Engineer, LSE Space GmBH Argelsrieder Feld 22, 82234 Wessling, Germany

II. Initial Uncertainty

Montenbruck and Gill describe the relationship between Cartesian and Keplerian uncertainties. For example, they describe how to compute the partials of the Cartesian position and velocity with respect to the semi-major axis. Their method starts with definition of the basis coordinates.

Given

$$\mathbf{D}_\Omega = \begin{bmatrix} \cos(\Omega) \\ \sin(\Omega) \\ 0 \end{bmatrix} \tag{1}$$

$$\mathbf{D}_\mathbf{N} = \begin{bmatrix} \sin(\Omega)\sin(i) \\ -\cos(\Omega)\sin(i) \\ \cos(i) \end{bmatrix} \tag{2}$$

Then letting

$$\mathbf{P} = \cos(\omega)\,\mathbf{D}_\Omega + \sin(\omega)\,(\mathbf{D}_\mathbf{N} \times \mathbf{D}_\Omega) \tag{3}$$

$$\mathbf{Q} = -\sin(\omega)\,\mathbf{D}_\Omega + \cos(\omega)\,(\mathbf{D}_\mathbf{N} \times \mathbf{D}_\Omega) \tag{4}$$

If we define the functions

$$x_{pq} = a\left(\cos(E) - e\right) \tag{5}$$

$$y_{pq} = a\sqrt{1-e^2}\cos(E) \tag{6}$$

$$x'_{pq} = -\frac{\sqrt{\mu a}}{r}\sin(E) \tag{7}$$

$$x'_{pq} = -\frac{\sqrt{\mu a}}{r}\sqrt{1-e^2}\cos(E) \tag{8}$$

Where

$$r = a\left(1 - e\cos(E)\right) \tag{9}$$

Then the partial of the position and velocity vector with respect to the semi-major axis is given by

$$\frac{\partial}{\partial a}\mathbf{R} = \frac{x_{pq}}{a}\mathbf{P} + \frac{y_{pq}}{a}\mathbf{Q} \tag{10}$$

$$\frac{\partial}{\partial a}\mathbf{V} = -\frac{x'_{pq}}{2a}\mathbf{P} - \frac{y'_{pq}}{2a}\mathbf{Q} \tag{11}$$

Montenbruck and Gill present similar equations for the partials of Cartesian position and velocity with respect to the remaining Keplerian elements. From these one can compose the complete matrix of partials.
Given

$$y \equiv \begin{pmatrix} r_x & r_y & r_z & v_x & v_y & v_z \end{pmatrix}^T \tag{12}$$

$$\alpha \equiv \begin{pmatrix} a & e & i & \Omega & \omega & \mathrm{M} \end{pmatrix}^T \tag{13}$$

Then the partial matrix is

$$\frac{\partial y}{\partial \alpha} = \begin{bmatrix} \frac{\partial r_x}{\partial a} & \frac{\partial r_x}{\partial e} & \frac{\partial r_x}{\partial i} & \frac{\partial r_x}{\partial \Omega} & \frac{\partial r_x}{\partial \omega} & \frac{\partial r_x}{\partial M} \\ \frac{\partial r_y}{\partial a} & \frac{\partial r_y}{\partial e} & \frac{\partial r_y}{\partial i} & \frac{\partial r_y}{\partial \Omega} & \frac{\partial r_y}{\partial \omega} & \frac{\partial r_y}{\partial M} \\ \frac{\partial r_z}{\partial a} & \frac{\partial r_z}{\partial e} & \frac{\partial r_z}{\partial i} & \frac{\partial r_z}{\partial \Omega} & \frac{\partial r_z}{\partial \omega} & \frac{\partial r_z}{\partial M} \\ \frac{\partial v_x}{\partial a} & \frac{\partial v_x}{\partial e} & \frac{\partial v_x}{\partial i} & \frac{\partial v_x}{\partial \Omega} & \frac{\partial v_x}{\partial \omega} & \frac{\partial v_x}{\partial M} \\ \frac{\partial v_y}{\partial a} & \frac{\partial v_y}{\partial e} & \frac{\partial v_y}{\partial i} & \frac{\partial v_y}{\partial \Omega} & \frac{\partial v_y}{\partial \omega} & \frac{\partial v_y}{\partial M} \\ \frac{\partial v_z}{\partial a} & \frac{\partial v_z}{\partial e} & \frac{\partial v_z}{\partial i} & \frac{\partial v_z}{\partial \Omega} & \frac{\partial v_z}{\partial \omega} & \frac{\partial v_z}{\partial M} \end{bmatrix} \tag{14}$$

In their method, the inverse partial uses Poisson brackets. Given

$$n = \sqrt{\frac{\mu}{a}} \tag{15}$$

Then the Poisson bracket coefficients are

$$P_{aM} = \frac{2}{n\, a} \tag{16}$$

$$P_{eM} = -\frac{(1-e)(1+e)}{n\, a^2 e} \tag{17}$$

$$P_{i\omega} = -\frac{1}{n\, a^2 \sqrt{1-e^2}\, \tan(i)} \tag{18}$$

$$P_{e\omega} = \frac{\sqrt{(1-e)(1+e)}}{n\, a^2 e} \tag{19}$$

$$P_{i\Omega} = -\frac{1}{n\,a^2\sqrt{1-e^2}\,\sin(i)} \tag{20}$$

The matrix representing the partial of the Keplerian elements with respect to the Cartesian elements is

$$\frac{\partial\alpha}{\partial y} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & P_{aM} \\ 0 & 0 & 0 & 0 & P_{e\omega} & P_{eM} \\ 0 & 0 & 0 & P_{i\Omega} & P_{i\omega} & 0 \\ 0 & 0 & -P_{i\Omega} & 0 & 0 & 0 \\ 0 & -P_{e\omega} & -P_{i\omega} & 0 & 0 & 0 \\ -P_{aM} & -P_{eM} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial v_x}{\partial a} & \frac{\partial v_y}{\partial a} & \frac{\partial v_z}{\partial a} & -\frac{\partial r_x}{\partial a} & -\frac{\partial r_y}{\partial a} & -\frac{\partial r_z}{\partial a} \\ \frac{\partial v_x}{\partial e} & \frac{\partial v_y}{\partial e} & \frac{\partial v_z}{\partial e} & -\frac{\partial r_x}{\partial e} & -\frac{\partial r_y}{\partial e} & -\frac{\partial r_z}{\partial e} \\ \frac{\partial v_x}{\partial i} & \frac{\partial v_y}{\partial i} & \frac{\partial v_z}{\partial i} & -\frac{\partial r_x}{\partial i} & -\frac{\partial r_y}{\partial i} & -\frac{\partial r_z}{\partial i} \\ \frac{\partial v_x}{\partial \Omega} & \frac{\partial v_y}{\partial W} & \frac{\partial v_z}{\partial \Omega} & -\frac{\partial r_x}{\partial \Omega} & -\frac{\partial r_y}{\partial \Omega} & -\frac{\partial r_z}{\partial \Omega} \\ \frac{\partial v_x}{\partial \omega} & \frac{\partial v_y}{\partial \omega} & \frac{\partial v_z}{\partial \omega} & -\frac{\partial r_x}{\partial \omega} & -\frac{\partial r_y}{\partial \omega} & -\frac{\partial r_z}{\partial \omega} \\ \frac{\partial v_x}{\partial M} & \frac{\partial v_y}{\partial M} & \frac{\partial v_z}{\partial M} & -\frac{\partial r_x}{\partial M} & -\frac{\partial r_y}{\partial M} & -\frac{\partial r_z}{\partial M} \end{bmatrix} \tag{21}$$

Finally, the Keplerian uncertainty for a given set of Keplerian elements and Cartesian uncertainty at a given epoch is computed by first computing the partials of the Keplerian with respect to the Cartesian elements using the Keplerian elements at the initial epoch

$$A_0 = \left.\frac{\delta\alpha}{dy}\right|_{t_0} \tag{22}$$

Then the Keplerian uncertainty is

$$\delta\,\alpha_0 = \begin{bmatrix} \delta a_0 \\ \delta e_0 \\ \delta i_0 \\ \delta\Omega_0 \\ \delta\omega_0 \\ \delta M_0 \end{bmatrix} = A_0 \cdot \begin{bmatrix} \delta r_x \\ \delta r_y \\ \delta r_z \\ \delta v_x \\ \delta v_y \\ \delta v_z \end{bmatrix} \tag{23}$$

III. Propagating Uncertainty To the Next Maneuver

From the initial uncertainty, one can assemble the initial covariance matrix

$$P_0 = \begin{bmatrix} {\sigma_{a.0}}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & {\sigma_{e.0}}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & {\sigma_{i.0}}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & {\sigma_{\Omega.0}}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & {\sigma_{\omega.0}}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & {\sigma_{M.0}}^2 \end{bmatrix} \tag{24}$$

To propagate the covariance, we need first to define a state transition matrix. The matrix for a simple Keplerian propagation is

$$\Phi_{kep}(a,t_0,t)=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -\frac{3}{2}\left(\frac{n}{a}(t-t_0)\right) & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{25}$$

Where

t_0 = Initial epoch

t = Arbitrary epoch

The state transition between two arbitrary is

$$\Phi_\alpha(t_2,t_1,t_0)=\Phi_{kep}(t_2,t_0)\Phi_{kep}(t_1,t_0)^{-1} \tag{26}$$

The propagated covariance is

$$P_2=\Phi_\alpha(t_2,t_1,t_0)\,P_1\,\Phi_\alpha(t_2,t_1,t_0)^T \tag{27}$$

Where

P_1 = Covariance at time t_1

P_2 = Covariance at time t_2

IV. Incorporating Maneuver Uncertainty

Bate, Mueller and White describe the change in semi-major axis due to a small Delta-V

$$\Delta a=\frac{2e\sin(f)}{n\sqrt{1-e^2}}\Delta v_r+\frac{2a\sqrt{1-e^2}}{n\,r}\Delta v_s \tag{28}$$

The differential with respect to the components of Delta-V can be derived analytically

$$\frac{\partial\Delta a}{\partial\Delta v_r}=\frac{2e\sin(f)}{n\sqrt{1-e^2}} \tag{29}$$

$$\frac{\partial\Delta a}{\partial\Delta v_s}=\frac{2a\sqrt{1-e^2}}{n\,r} \tag{30}$$

$$\frac{\partial\Delta a}{\partial\Delta v_w}=0 \tag{31}$$

Bate, Mueller and White supply similar expression of the change in each Keplerian element as a function of the Delta-V delivered in each direction. Using these, one can assemble the Jacobian

$$H = \begin{bmatrix} \frac{\partial a}{\partial \Delta v_r} & \frac{\partial e}{\partial \Delta v_r} & \frac{\partial i}{\partial \Delta v_r} & \frac{\partial \Omega}{\partial \Delta v_r} & \frac{\partial \omega}{\partial \Delta v_r} & \frac{\partial M}{\partial \Delta v_r} \\ \frac{\partial a}{\partial \Delta v_s} & \frac{\partial e}{\partial \Delta v_s} & \frac{\partial i}{\partial \Delta v_s} & \frac{\partial \Omega}{\partial \Delta v_s} & \frac{\partial \omega}{\partial \Delta v_s} & \frac{\partial M}{\partial \Delta v_s} \\ \frac{\partial a}{\partial \Delta v_w} & \frac{\partial e}{\partial \Delta v_w} & \frac{\partial i}{\partial \Delta v_w} & \frac{\partial \Omega}{\partial \Delta v_w} & \frac{\partial \omega}{\partial \Delta v_w} & \frac{\partial M}{\partial \Delta v_w} \end{bmatrix} \tag{32}$$

So, letting

$\Delta v_r, \delta\, \Delta v_w$ = Uncertainty in Delta V in the radial and normal directions
$\delta\, \Delta v_s$ = Uncertainty in Delta V in the direction formed by the cross product of the radial and normal vectors

Then we can assemble the matrix

$$W = \begin{bmatrix} (\delta \Delta v_r)^2 & 0 & 0 \\ 0 & (\delta \Delta v_s)^2 & 0 \\ 0 & 0 & (\delta \Delta v_w)^2 \end{bmatrix} \tag{33}$$

The covariance matrix will be increased by the uncertainty in the burn. After the burn, the covariance becomes

$$P_{postburn} = P_{preburn} + H^T W\, H \tag{34}$$

V. Converting Uncertainty into Usable Information

So far, all we've done is establish the evolution of the uncertainty in the Keplerian elements. We must convert this into actionable information. For example, in Geosynchronous Transfer Orbits (GTO), one is interested in the Earth longitude and the orbital radius where each burn will take place. The longitude is given by

$$\lambda(t, \mathbf{R}) = angle\left(r_x, r_y\right) - GHA(t) \tag{35}$$

Where

$GHA(t)$ = Greenwich Hour Angle at the time t
$angle(x, y)$ = Angle whose cosine is x and sine is y

In this case, the position of the spacecraft can be expressed as a function of the Keplerian elements:

$$\mathbf{R}(a, e, i, \Omega, \omega, f) = r(a, e, f)\left(\cos(f)\,\mathbf{P}(i, \Omega, \omega) + \sin(f)\,\mathbf{Q}(i, \Omega, \omega)\right) \tag{36}$$

One approach would be to analytically compute the differential relationship between longitude and the Keplerian elements. However it is worthwhile to try out numerical differentiation. For example, given

$\delta\delta a$ = Small differential variation in semi-major axis. Say about 5 meters.

Then

$$\frac{\partial \lambda}{\partial a} = \frac{\lambda\left(t, \mathbf{R}\left(a + \delta\delta a, e, i, \Omega, \omega, f\right)\right) - \lambda\left(t, \mathbf{R}\left(a - \delta\delta a, e, i, \Omega, \omega, f\right)\right)}{2\,\delta\delta a} \tag{37}$$

One can construct similar expressions for the partial of the longitude with respect to each of the other Keplerian elements. From this, one can assemble the following differential matrix

$$\frac{\partial \lambda}{\partial \alpha} = \begin{bmatrix} \frac{\partial \lambda}{\partial a} & \frac{\partial \lambda}{\partial e} & \frac{\partial \lambda}{\partial i} & \frac{\partial \lambda}{\partial \Omega} & \frac{\partial \lambda}{\partial \omega} & \frac{\partial \lambda}{\partial f} \end{bmatrix} \tag{38}$$

In the case of the orbital radius, we have an analytic expression that is easy to differentiate, but whether one works this out our uses numerical differentiation, one still arrives at:

$$\frac{\partial r}{\partial \alpha} = \begin{bmatrix} \frac{\partial r}{\partial a} & \frac{\partial r}{\partial e} & 0 & 0 & 0 & \frac{\partial r}{\partial f} \end{bmatrix} \tag{39}$$

Ultimately, we want plots of Earth longitude vs. orbital radius. We will superimpose the uncertainty on these plots. We need to have functions that compute little ellipsoids of uncertainty in longitude an orbit radius around each plotted value of nominal longitude and radius. The process starts with the matrix

$$\frac{\partial U}{\partial \alpha} = \begin{bmatrix} \frac{\partial r}{\partial a} & \frac{\partial r}{\partial e} & 0 & 0 & 0 & \frac{\partial r}{\partial f} \\ \frac{\partial \lambda}{\partial a} & \frac{\partial \lambda}{\partial e} & \frac{\partial \lambda}{\partial i} & \frac{\partial \lambda}{\partial \Omega} & \frac{\partial \lambda}{\partial \omega} & \frac{\partial \lambda}{\partial f} \end{bmatrix} \tag{40}$$

Next one must compute the uncertainty associated with this matrix

$$\delta U = \frac{\partial U}{\partial \alpha} \delta \alpha \tag{41}$$

Where

$\delta\alpha \equiv \left(\delta a \quad \delta e \quad \delta i \quad \delta\Omega \quad \delta\omega \quad \delta \mathrm{M}\right)^T$ = Column vector representing the uncertainty of each of the Keplerian elements

The ellipsoids of uncertainty around each value of longitude and orbital radius do not describe a perfect circle. The uncertainty in longitude might for example be much bigger than that in orbital radius. The uncertainty ellipsoid would be elliptical. More interesting, the axes of the ellipse need not be aligned with the axes of the plot. It can be tilted a little bit. To handle this, we need first to know where the long and short axes of the ellipse points. Since δU is a 2×2 matrix, there will be two eigen-values and two eigenvectors

Given

u_1^{eigen}, u_2^{eigen} = Eigen-values of the matrix δU

$\mathbf{U}_1^{eigen}, \mathbf{U}_2^{eigen}$ = Corresponding eigen-vectors of the matrix δU

Then the axes of the error ellipsoids are

$$\mathbf{E}_1 = \sqrt{u_1^{eigen}}\, \mathbf{U}_1^{eigen} \tag{42}$$

$$\mathbf{E}_2 = \sqrt{u_2^{eigen}}\,\mathbf{U}_2^{eigen} \tag{43}$$

Note that each is a vector with two elements

$$\mathbf{E}_1 = \begin{bmatrix} E_1^r \\ E_1^\lambda \end{bmatrix} \tag{44}$$

$$\mathbf{E}_2 = \begin{bmatrix} E_2^r \\ E_2^\lambda \end{bmatrix} \tag{45}$$

For a given point along the ephemeris, use the Keplerian data to compute the orbital radius and longitude at that epoch. Also, compute the error eigen-axes and eigen-values at that point. To describe an ellipsoid about this point, one proceeds as follows

Given

r_n, λ_n = Orbital radius and Earth longitude at time t_n

Then if we want to describe an ellipsoid with points 360 deg apart then we could let, for example

$$j = 0 \; \square \; 360 \qquad \text{and} \qquad \Delta\theta = 1^{\circ}$$

Letting

$$\theta_j = j\,\Delta\theta \tag{46}$$

Then the distance from r_n, λ_n to each point on the ellipse is given by

$$\delta r_j = \cos(\theta_j) E_1^r + \sin(\theta_j) E_2^r \tag{47}$$

$$\delta \lambda_j = \cos(\theta_j) E_1^\lambda + \sin(\theta_j) E_2^\lambda \tag{48}$$

VI. Example

Consider a spacecraft in near-geosynchronous drift orbit. It is a few degrees West of station longitude. It is drifting toward station at a few degrees per day. The plan is to execute a few burns to shape the eccentricity, arrest drift and arrive at station. Along the path to station, the spacecraft passes close to several operational spacecraft. There is an initial trajectory uncertainty, as well as uncertainty about the planned burns. This example investigates two questions. Will the trajectory veer too close to operational spacecraft? Will the spacecraft arrive at station longitude or will it miss?

A. Initial Trajectory and Preliminary Plan

The initial trajectory that we'll use for this example is as follows

Table 1. Initial State Vector

Parameter	Value	Parameter	Value
Epoch	2010/07/29 08:15:00.000	Ω_0 (deg)	329.9011
a_0 (km)	42083.2515	ω_0 (deg)	182.4479
e_0	0.002	f_0 (deg)	7.7736
i_0 (deg)	0.0425	λ_0 (deg)	89.4958

The uncertainty in these elements happens to be expressed in Cartesian coordinates. Note that while these are 3 sigma values, it is a matter of taste; if one uses 3 sigma inputs, one obtains 3 sigma outputs.

Table 2. Initial State Vector Uncertainty

Parameter	Value	Parameter	Value
X Position Uncertainty (m)	400	X Velocity Uncertainty (m/s)	0.04
Y Position Uncertainty (m)	400	Y Velocity Uncertainty (m/s)	0.04
Z Position Uncertainty (m)	400	Z Velocity Uncertainty (m/s)	0.04

The timing and size of the planned burns is as follows:

Table 3. Preliminary Burn Plan

Epoch	Event	Tangential Delta V (m/s)	Normal Delta V (m/s)	Radial Delta V (m/s)
2010/08/01 00:15:00.000	BURN 1	0.9	-0.045	0
2010/08/02 19:00:00.000	BURN 2	2.2	-0.115	0

The uncertainty associated with each burn is:

Table 4. Preliminary Burn Plan Uncertainty

Tangential Delta V Uncertainty	Normal Delta V Uncertainty	Radial Delta V Uncertainty
1%	1%	5%

B. Analysis of the Preliminary Plan

Now we can apply the techniques under discussion. First, convert the initial trajectory uncertainty from Cartesian to the corresponding uncertainty in the Keplerian elements:

Table 5. Keplerian Trajectory Uncertainty

Uncertainty	Value	Uncertainty	Value
δa_0 (km)	1.886851	$\delta \Omega_0$ (deg)	0.545382
δe_0	0.000041	$\delta \omega_0$ (deg)	0.133874
δi_0 (deg)	0.000827	δf_0 (deg)	0.409915

The nominal trajectory is an output of whatever propagator is currently in use, it is not computed by the techniques presented in this paper. Figure 1 presents that trajectory in terms of the evolution longitude and the orbital radius. Using the techniques described here, uncertainty ellipsoids have been drawn at regular intervals along that trajectory.

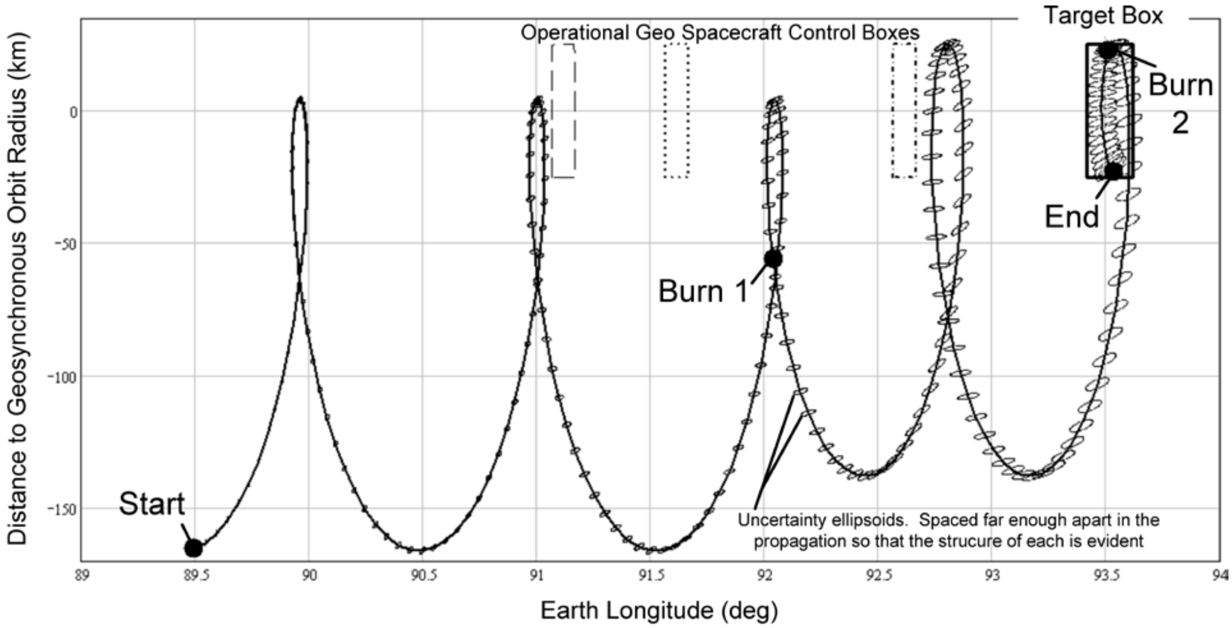

Figure 1. Preliminary Plan

Figure 1 shows that while the nominal trajectory arrives inside the box, portions of the uncertainty fall outside target. This demonstrates it is not quite time to commit to this burn plan. Fortunately, the same plot presents the means to address the problem: the uncertainty in the trajectory leading up to the first burn is still quite high. This must be hammered down. However, this plot was generated some days before the coming burn. There is time to improve the determination of the orbit and re-design the burns.

C. Updated Trajectory and Re-planning

Two days later, more tracking data has been collected. Routine orbit determination shows that the trajectory has slightly changed.

Table 6. Improved Trajectory

Parameter	Value	Parameter	Value
Epoch	2010/07/31 03:00:00.000	Ω_0 (deg)	340.5241
a_0 (km)	42083.6596	ω_0 (deg)	169.7228
e_0	0.002	f_0 (deg)	294.5449
i_0 (deg)	0.044	λ_0 (deg)	91.1593

Also, one naturally has revised nominal burn plan to account for the slightly different nominal trajectory.

Table 7. Improved Plan for Burns 1 and 2

Epoch	Event	Tangential Delta V (m/s)	Normal Delta V (m/s)	Radial Delta V (m/s)
2010/08/01 00:15:00.000	BURN 1	0.9	-0.045	0.483
2010/08/02 19:00:00.000	BURN 2	2	-0.115	1.235

Because of the additional tracking data, the quality of the orbit determination uncertainty is much improved.

Table 8. Improved Cartesian Trajectory Uncertainty

Parameter	Value	Parameter	Value
X Position Uncertainty (m)	6	X Velocity Uncertainty (m/s)	0.00009
Y Position Uncertainty (m)	4	Y Velocity Uncertainty (m/s)	0.00052
Z Position Uncertainty (m)	4	Z Velocity Uncertainty (m/s)	0.00007

D. Analysis of the Updated Trajectory and re-plan

Now we can again apply the techniques of this paper. The corresponding uncertainty in the Keplerian elements is also substantially improved:

Table 9. Improved Keplerian Trajectory Uncertainty

Uncertainty	Value	Uncertainty	Value
δa_0 (km)	0.007799	$\delta \Omega_0$ (deg)	0.003404
δe_0	0	$\delta \omega_0$ (deg)	0.005894
δi_0 (deg)	0.000005	δf_0 (deg)	0.002478

Look at Figure 2. The revised propagation still arrives in the target box. Much more important however is that the uncertainty ellipsoids drawn around this trajectory are also within the box. You are ready to commit to the first burn.

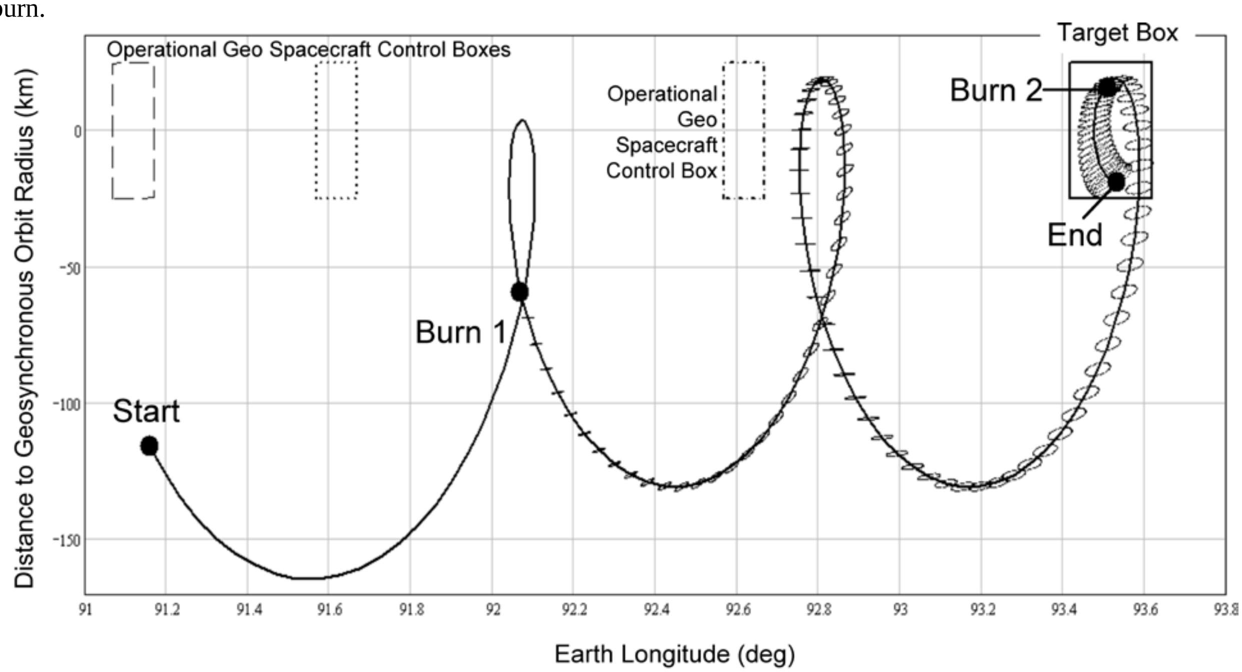

Figure 2. Re-planning Prior to Burn 1

E. Re-Plan Prior to Burn 2

After the first burn has executed, more tracking data is accumulated and the trajectory is re-estimated

Table 10. Pre-Burn 2 Trajectory

Parameter	Value	Parameter	Value
Epoch	2010/08/02 08:53:10.000	Ω_0 (deg)	350.2618
a_0 (km)	42109.9938	ω_0 (deg)	147.129
e_0	0.0018	f_0 (deg)	40.1475
i_0 (deg)	0.0462	λ_0 (deg)	93.4011

Also, burn 2 is revised to account for the slightly different nominal trajectory and results of engine calibration.

Table 11. Improved Plan for Burn 2

Epoch	Event	Tangential Delta V (m/s)	Normal Delta V (m/s)	Radial Delta V (m/s)
2010/08/02 19:00:00.000	BURN 2	2.05	-0.113	1.208

F. Analysis of the Uncertainty for the Revised Plan for Burn 2

Now we can apply the techniques of this paper to the re-plan. The Keplerian element uncertainty from orbit determination is shown in Table 12.

Table 12. Trajectory Uncertainty Prior to Burn 2

Uncertainty	Value	Uncertainty	Value
δa_0 (km)	0.044308	$\delta \Omega_0$ (deg)	0
δe_0	0.0000003	$\delta \omega_0$ (deg)	0
δi_0 (deg)	0.0000076	δf_0 (deg)	0.000007

The revised propagation now arrives well within the box:

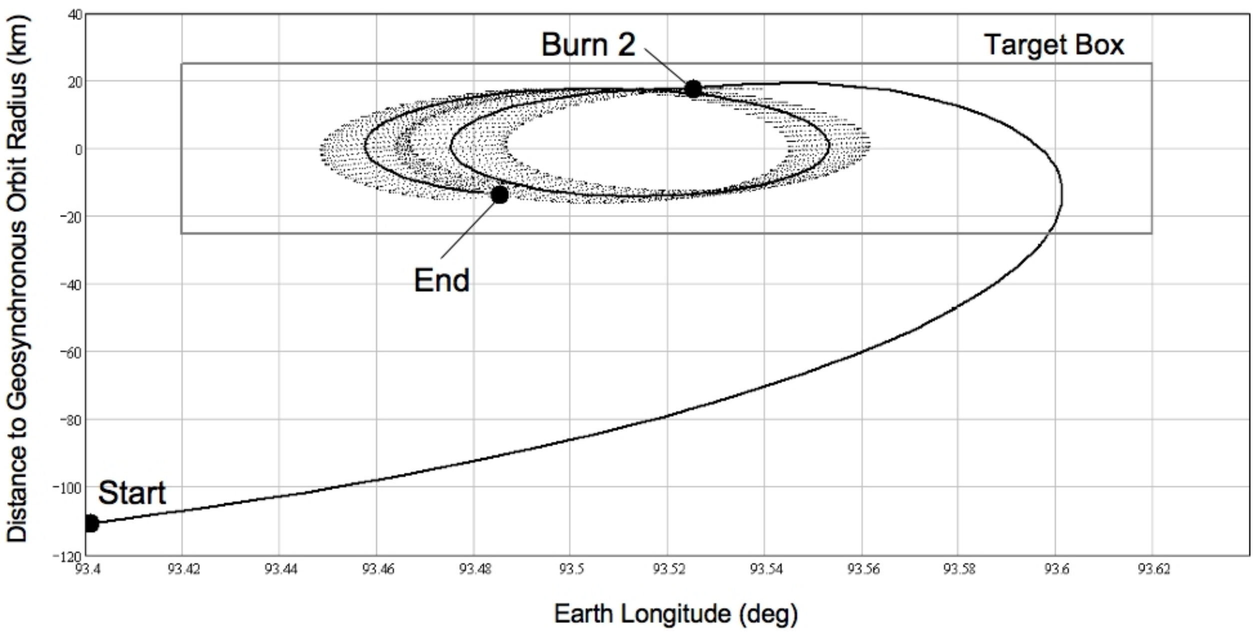

Figure 3. Orbit Determination Prior to Burn 2

Figure 3 demonstrates that burn 2 is robust. The entire trajectory and uncertainty is contained within the box. The spacecraft will make it to station.

VII. Conclusion

We have discussed a methodology that allows uncertainty to be incorporated into the mission planning process in a meaningful and immediate way. The technique uses differentials to relate uncertainties in trajectory, maneuver and attitude to corresponding uncertainties in target mission parameters. Provided the input errors are reasonably small, computed output uncertainties will reflect the uncertainty in achieving mission targets.

References

[1]Montenbruck, O. and Gill, E., *Satellite Orbits: Models Methods, Applications*, Copyright 2000 by Springer- Verlag ISBN 3-540-67280-X.

[2]Bate, R.R, Mueller, D.D., White, J.E., *Fundamentals of Astrodynamics*, Copyright 1971 by Dover Publications Inc., ISBN 0-486-60061-0.