Profile Optimization of Satellite Antenna for Angular Jerk Minimization

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This paper presents the optimization of the satellite antenna profile (SAP) to minimize the angular jerk which is the time derivative of the angular acceleration. The method of moving asymptotes (MMA), which is a gradient-based optimization algorithm, is employed to solve the optimization problem. The sequential angle of the SAP is defined as the design variable for the optimization. The off-pointing margin angle, which is the maximum allowable range of the satellite antenna rotation for the communication with the ground station, is used for the side limits of the design variable. Three constraint sets with one objective function are formulated as the optimization problem. The objective function is the total sum of the squared angular jerk. The first set of constraints is the angular velocity and acceleration, the second is the angular jerk alone, and the third is the angular velocity, acceleration, and jerk. In numerical examples, two real SAPs are used for implementing the proposed optimization algorithm. The optimization results show the effectiveness of the proposed algorithm with great reduction of the angular jerk. The objective function and the computation time for the three sets of the optimization problems are compared and discussed.

I. Introduction

KOREA multi-purpose satellite 3 (KOMPSAT-3), orbiting 685 km above the earth, is maneuvered to take very high-resolution ground images using a multispectral camera (MSC). The accuracy of the MSC is 0.7 m of panchromatic and 3.2 m of multispectral images. An image taken by the MSC is transmitted to the ground station using an X-band antenna. The X-band antenna can rotates in two directions of azimuth and elevation angle in order to point toward the ground station antenna as tracking the satellite antenna profile (SAP). The off-pointing margin boundary is the maximum range of the antenna pointing error for communication. The SAP and the off-pointing margin boundary consist of sequential angles with a uniform sampling time. Therefore, the X-band antenna must reach each sequential angle within the off-pointing margin angle on the uniform sampling time for communication. Reference 1 presented a detailed procedure of scheduling the SAP and employed a reinforcement learning approach to satisfy the off-pointing margin boundary and the mechanical constraints of the antenna rotation velocity and acceleration. During the rotation of the X-band antenna, undesirable vibration can arise and affect the satellite body. Vibration of the satellite body causes degradation of the high-resolution image quality and motion control ability. The angular jerk, which is the third derivative of rotation angle with respect to time, induces transient vibration and

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affects the stabilization of the satellite. Therefore, the SAP must be optimized to minimize the angular jerk so that the transient vibration is reduced and satellite control accuracy is increased.

The two broad categories of direct or search methods and indirect or optimality criteria methods are most widely used for profile optimization.[2] Based on the two key drawbacks of both methods, namely, the lack of a global search capability and the requirement of a suitable initial guess, a hybrid method combining a gradient-based method and a global search method, such as a genetic algorithm (GA), has been studied.[3-6] Although the GA is not computationally competitive against gradient-based methods, it is useful in finding the approximate initial guess of the optimal solution due to global search compatibility. Optimization of the profile with restriction of jerk has been reported in several papers. The polyhedron search algorithm is applied to the optimization of path planning using the cubic spline functions to minimize the traveling time of robot manipulators with joint velocity, acceleration, and jerk constraints.[7-9] Since optimization of the cubic spline using the polyhedron search algorithm yields a local optimal solution, algorithms for finding a global optimal solution are presented. Methods based on interval analysis (IA) and a GA to minimize the total travelling time subject to constraints on joint velocities, acceleration, and jerk were proposed,[10,11] and a hybrid optimization method combining the GA and sequential quadratic programming (SQP) was proposed.[12] Unlike the use of jerk for a constraint, the integral of the squared jerk is applied for the objective function,[13] and the maximum value of the jerk is minimized using the minimax approach.[14]

In previous studies, there has been no publication covering the optimization of the SAP to minimize or reduce the angular jerk. This paper introduces the minimization of the angular jerk of the SAP using the gradient-based optimization method. Without the use of a global search algorithm to find the initial guess, the initial value of the design variable is defined as the SAP which is generated by a geometrical relationship between the satellite and targeted ground station. The sequential angle of the SAP is defined as the design variable. The off-pointing margin angle is used for the side limit of the design variable. The design variable is updated within the side limit during optimization. The angular velocity, acceleration, and jerk are determined by the finite difference method (FDM). The objective function is the total sum of the squared angular jerk, and three sets of constraints are formed. The first set of constraints is the angular velocity and acceleration, the second is the angular jerk alone, and the third is the angular velocity, acceleration, and jerk. The method of moving asymptotes (MMA),[15,16] which was introduced by Svanberg in 1987, is used to solve the optimization problems. The MMA is a gradient-based optimization algorithm which has been used in many fields.[17] Gradient-based optimization schemes require design sensitivity analysis (DSA), which is performed by the derivatives of the objective function and the constraints with respect to the design variable.[18] Since the objective function and the constraints are explicitly expressed by the design variable, the derivatives are easily computed in this paper. The proposed method is applied to the numerical examples of two real SAPs, and then the optimization results from three optimization problems are discussed.

The remainder of this paper is organized into the following sections. Section 2 describes the SAP and the off-pointing margin boundary. In Section 3.1, the optimization problems in terms of the design variable are formulated. The design sensitivity is derived in Section 3.2. In Section 4, numerical steps of the optimization are explained. Section 4.1 and 4.2 show the optimization of two numerical examples and compare the optimization results of three optimization problems. Finally, conclusions are given in Section 5.

II. Satellite Antenna Profile

The X-band antenna rotates to both angles of the azimuth and the elevation to direct to the ground station antenna for data communication. Fig. 1 schematically shows the azimuth over elevation system of the X-band antenna. The rotation ranges of the azimuth and elevation angle are from 0 degree to 360 degree and from 14.8 degree to 145 degree, respectively. The initial azimuth and elevation angles are calculated from the relationship of the satellite attitude according to the earth-centered fixed (ECF) frame, the orientation of antenna in the body frame, satellite position according to ECF, the position of targeted ground station according to ECF, and the position of projected nadir point.[1] The central axis of the satellite and the ground station antennas are matched at the initial angles. The maximum allowable rotation angle for the satellite antenna to communicate with the ground station is called the off-pointing margin angle. The azimuth and the elevation angles can have any value within the off-pointing margin angle for communication. The SAP and the off-pointing margin angle are described in Fig. 2. Fig. 3 shows the SAP of the azimuth and elevation angle.

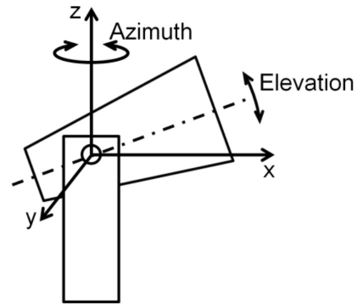
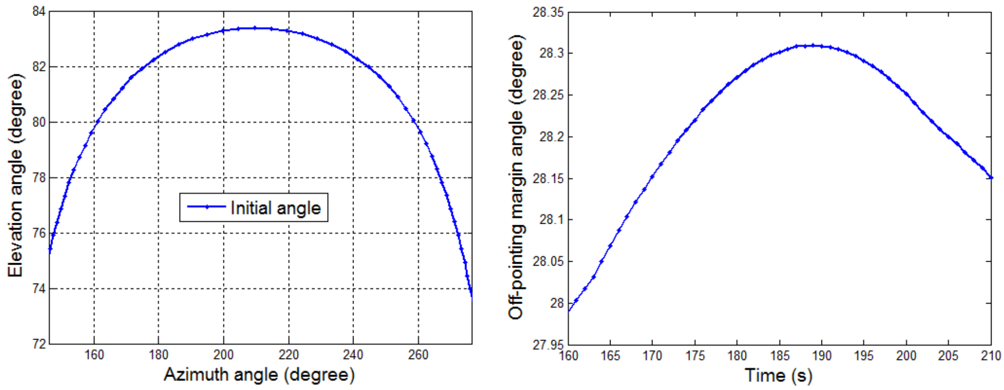
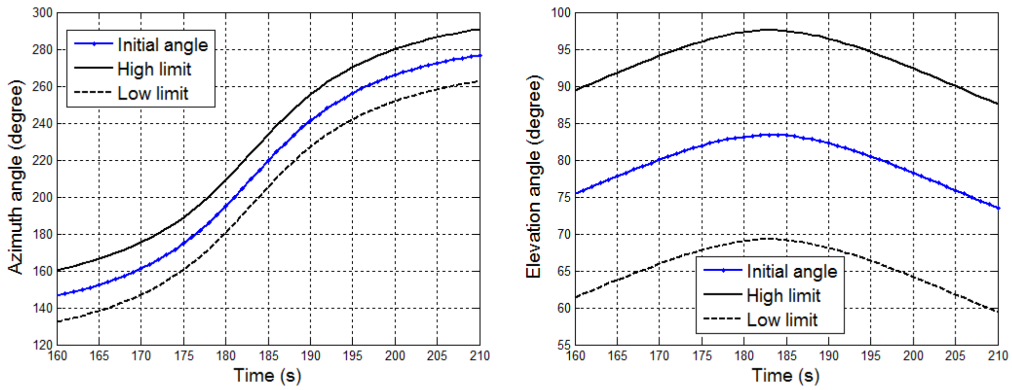


Figure 1. Diagrammatic representation of the rotation of the X-band antenna.



(a) (b)

Figure 2. SAP: (a) initial angle, and (b) off-pointing margin angle.



(a) (b)

Figure 3. Desired angle and off-pointing margin angle: (a) azimuth angle w.r.t time, and (b) elevation angle w.r.t time.

III. Formulation of Optimization Problem

The formulation of the optimization problem, which influences the final goal of the optimization, is sometimes tricky. The following three steps are required to formulate the optimization problem.[19] The first step is the identification of the design variable. The second is the identification of the objective function in terms of the design variable. The third is the identification of the constraints in terms of the design variable.

The identification of the design variable is often the most difficult part of the entire optimization procedure. In this paper, the sequential angle of the SAP, $\theta_1, \theta_2, \theta_3, \cdots, \theta_{n-1}, \theta_n$, is defined as the design variable. Here, n is the number of design variables. The angular velocity $\dot{\theta}$, the angular acceleration $\ddot{\theta}$, and the angular jerk $\dddot{\theta}$ are the first, second, and third derivatives of the angle with respect to time, which are calculated as follows:

$$\dot{\theta}_1 = \frac{\theta_2 - \theta_1}{\Delta t}, \dot{\theta}_2 = \frac{\theta_3 - \theta_2}{\Delta t}, \cdots, \dot{\theta}_{n-1} = \frac{\theta_n - \theta_{n-1}}{\Delta t} \tag{1}$$

$$\ddot{\theta}_1 = \frac{\dot{\theta}_2 - \dot{\theta}_1}{\Delta t}, \ddot{\theta}_2 = \frac{\dot{\theta}_3 - \dot{\theta}_2}{\Delta t}, \cdots, \ddot{\theta}_{n-2} = \frac{\dot{\theta}_{n-1} - \dot{\theta}_{n-2}}{\Delta t} \tag{2}$$

$$\dddot{\theta}_1 = \frac{\ddot{\theta}_2 - \ddot{\theta}_1}{\Delta t}, \dddot{\theta}_2 = \frac{\ddot{\theta}_3 - \ddot{\theta}_2}{\Delta t}, \cdots, \dddot{\theta}_{n-3} = \frac{\ddot{\theta}_{n-2} - \ddot{\theta}_{n-3}}{\Delta t} \tag{3}$$

where Δt is the sampling time.

A. Optimization Problem

The angular jerk is used for the function of quantifying the smoothness of the profile.[20] Since this paper aims to minimize the angular jerk to reduce the vibration of the satellite antenna, the total sum of the squared angular jerk is considered for the objective function. The high angular jerk is critical to satellite vibration, whereas the low one is trivial. The large angular jerk becomes dominant in the objective function through taking the square as follows:

$$\text{Minimize} \quad f_0(\theta) = \sum_{i=1}^{n-3} \dddot{\theta}_i^2 \tag{4}$$

Three sets of constraints are applied for the optimization problem. The first set of constraints includes both the angular velocity and the angular acceleration. The second is the angular jerk. The third is the angular velocity, the angular acceleration, and the angular jerk. The angular velocity and acceleration are respectively restricted by $\dot{\theta}_{\max}$ of 6 degree/s and $\ddot{\theta}_{\max}$ of 2 degree/s^2 due to disturbance by the angular momentum of the antenna and due to the limited power of the antenna pointing system (APS). The constraint of the maximum angular jerk $\dddot{\theta}_{\max}$ is defined as 10 % of the maximum jerk of the initial SAP. Three sets of constraints are given as follows:

Set 1:

$$\begin{aligned} \text{subject to} \quad -\dot{\theta}_{\max} \le \dot{\theta}_i \le \dot{\theta}_{\max} \quad & \text{for} \quad i = 1,\ldots,n-1 \\ -\ddot{\theta}_{\max} \le \ddot{\theta}_i \le \ddot{\theta}_{\max} \quad & \text{for} \quad i = 1,\ldots,n-2 \end{aligned} \tag{5}$$

Set 2:

$$\text{subject to} \quad -\dddot{\theta}_{\max} \le \dddot{\theta}_i \le \dddot{\theta}_{\max} \quad \text{for} \quad i = 1,\ldots,n-3 \tag{6}$$

Set 3:

$$\begin{aligned} \text{subject to} \quad -\dot{\theta}_{\max} \le \dot{\theta}_i \le \dot{\theta}_{\max} \quad & \text{for} \quad i = 1,\ldots,n-1 \\ -\ddot{\theta}_{\max} \le \ddot{\theta}_i \le \ddot{\theta}_{\max} \quad & \text{for} \quad i = 1,\ldots,n-2 \\ -\dddot{\theta}_{\max} \le \dddot{\theta}_i \le \dddot{\theta}_{\max} \quad & \text{for} \quad i = 1,\ldots,n-3 \end{aligned} \tag{7}$$

The side limit of the design variable is obtained as follows:

$$\theta_i - \frac{\Delta\theta_i}{2} \le \theta_i \le \theta_i + \frac{\Delta\theta_i}{2} \quad \text{for} \ i = 1,2,\ldots,n \tag{8}$$

where $\Delta\theta_i$ is the off-pointing margin angle of i^{th} design variable.

B. Design Sensitivity Analysis

The DSA is required to determine the direction of desirable design change to improve the performance measures of the system in the gradient-based optimization method. Design sensitivity is calculated by the differentiation of the objective function and the constraint function with respect to the design variable. The design sensitivity of the objective function in Eq. (4) is calculated as

$$\frac{df_0}{d\theta_j} = \frac{d\left(\sum_{i=1}^{n-3} \dddot{\theta}_i^2\right)}{d\theta_j} = \frac{1}{\Delta t^6} \cdot \frac{d\left[\sum_{i=1}^{n-3} \left(\theta_{i+3} - 3\theta_{i+2} + 3\theta_{i+1} - \theta_i\right)^2\right]}{d\theta_j} \quad \text{for} \ \ j = 1,2,\ldots,n \tag{9}$$

The design sensitivities of the angular velocity, acceleration, and jerk in Eqs. (5)-(7) are calculated as

$$\frac{d\dot{\theta}_i}{d\theta_j} = \frac{1}{\Delta t} \cdot \frac{d\left(\theta_{i+1} - \theta_i\right)}{d\theta_j} \quad \text{for} \ \ j = 1,2,\ldots,n \tag{10}$$

$$\frac{d\ddot{\theta}_i}{d\theta_j} = \frac{1}{\Delta t^2} \cdot \frac{d\left(\theta_{i+2} - 2\theta_{i+1} + \theta_i\right)}{d\theta_j} \qquad \text{for} \quad j = 1, 2, \ldots, n \tag{11}$$

$$\frac{d\dddot{\theta}_i}{d\theta_j} = \frac{1}{\Delta t^3} \cdot \frac{d\left(\theta_{i+3} - 3\theta_{i+2} + 3\theta_{i+1} - \theta_i\right)}{d\theta_j} \qquad \text{for} \quad j = 1, 2, \ldots, n \tag{12}$$

IV. Numerical Implementation

The MMA is used to solve the optimization problem of minimizing the angular jerk. The MMA uses a special type of convex approximation. For each step of the iterative process, a strictly convex approximating subproblem is generated and solved. The generation of these subproblems is controlled by the so-called moving asymptotes, which both stabilize and speed up the convergence of the general process.[16] The gradient-based optimization of the SAP using MMA is carried out as follows:

Step 0. Choose an initial design variable.
Step 1. Evaluate the objective function and constraints in Eqs. (4)-(7).
Step 2. Calculate the gradients of the objective function and constraints in Eqs. (9)-(12).
Step 3. Optimize using MMA.
Step 4. Update the design variable.
Step 5. Check if the convergence test is satisfied. If not, go to step 1.

In step 0, the desired angle of the SAP is used for the initial design variable. An absolute tolerance of the maximum change of the design variables is typically used for the convergence test. In this paper, the iteration number is used for the convergence criteria to compare the time and the performance of using three sets in Table 1. The defined iteration number for terminating the optimization loop is 50.

As the numerical implementation for our proposed optimization method, we employ two examples of actual flight data. In sections 4.1 and 4.2, the optimization results according to three sets of optimization problems are compared to the initial values of the SAP. The computation time for the optimization, which needs to be reduced, is an important factor in evaluating the optimization performance and in scheduling the satellite operation. The computation times of the three sets are compared in addition to the objective function and constraints. The computer used to calculate the numerical examples was an Intel Core2 Quad CPU @ 2.5 GHz with 2 GB RAM, and the optimization program was developed by Matlab.

A. Example 1: General Case

The number of angles for the optimization is 280, and the sampling time is 1 s. The maximum angular velocities of the initial azimuth and elevation profile are 7.4685 degree/s and 1.1713 degree/s, respectively. The maximum angular accelerations of the initial azimuth and elevation profile are 1.4256 degree/s^2 and 0.2011 degree/s^2, respectively. 10 % of the maximum angular jerk of the initial azimuth and elevation profile are respectively 0.1084 degree/s^3 and 0.0167 degree/s^3, which are applied for the constraint of the angular jerk in Eqs. (6) and (7). The azimuth angular velocity violates the mechanical constraint of 6 degree/s.

Tables 1 and 2 summarize the optimization results using three sets of optimization problems. All constraints of the three sets are satisfied. The optimization results of set 1 are considerably worse than the results of the others, especially in the reduction of the objective function. As seen in the comparison of sets 2 and 3 in Tables 1 and 2, set 2 is more effective than set 3. Although the optimization results of both sets are almost the same, the computation time of set 2 is significantly less than that of set 3 because of the number of constraints.

Table 1. Optimization results of azimuth profile

Comparison criteria	Set 1	Set 2	Set 3
Reduction ratio of objective function (%)	2.1895	91.5472	92.9144
Maximum velocity (initial value: 7.4685) (degree/s)	6.0000	5.5642	5.2461
Maximum acceleration (initial value: 1.4256) (degree/s^2)	1.5723	0.7778	0.7231
Maximum jerk (initial value: 1.0848) (degree/s^3)	0.7675	0.1017	0.1009
Total time (s)	171	103	311

Table 2. Optimization results of elevation profile

Comparison criteria	Set 1	Set 2	Set 3
Reduction ratio of objective function (%)	10.4689	95.8858	96.0523
Maximum velocity (initial value: 1.1713) (degree/s)	1.1507	0.7147	0.7032
Maximum acceleration (initial value: 0.2011) (degree/s^2)	0.1900	0.0862	0.0845
Maximum jerk (initial value: 0.1672) (degree/s^3)	0.1444	0.0114	0.0111
Total time (s)	67	62	164

Fig. 4 shows the convergence histories of optimizing the azimuth and elevation profiles using set 2.

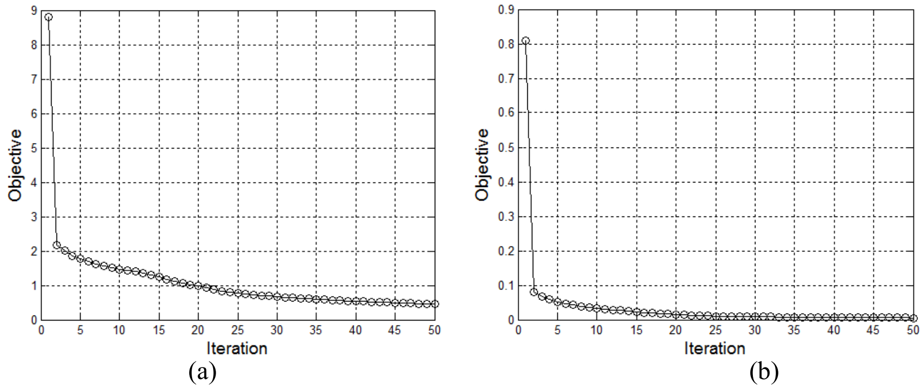
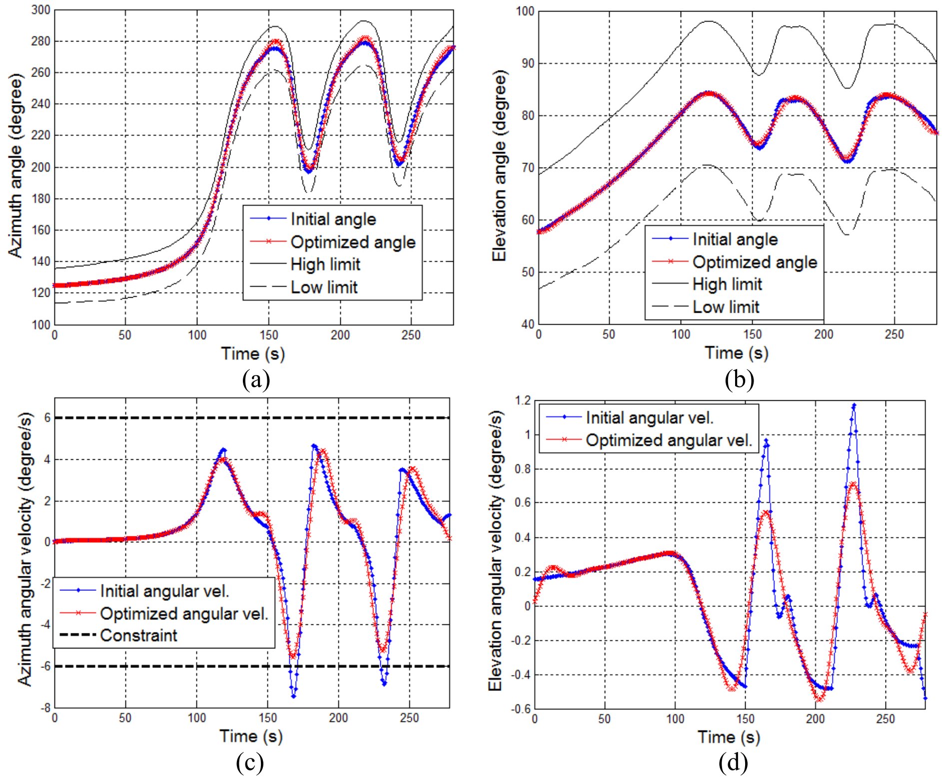


Figure 4. Optimization history of set 2: (a) azimuth angle, (b) elevation angle.

Fig. 5 shows the optimized results using set 2 compared to the initial values of the SAP.



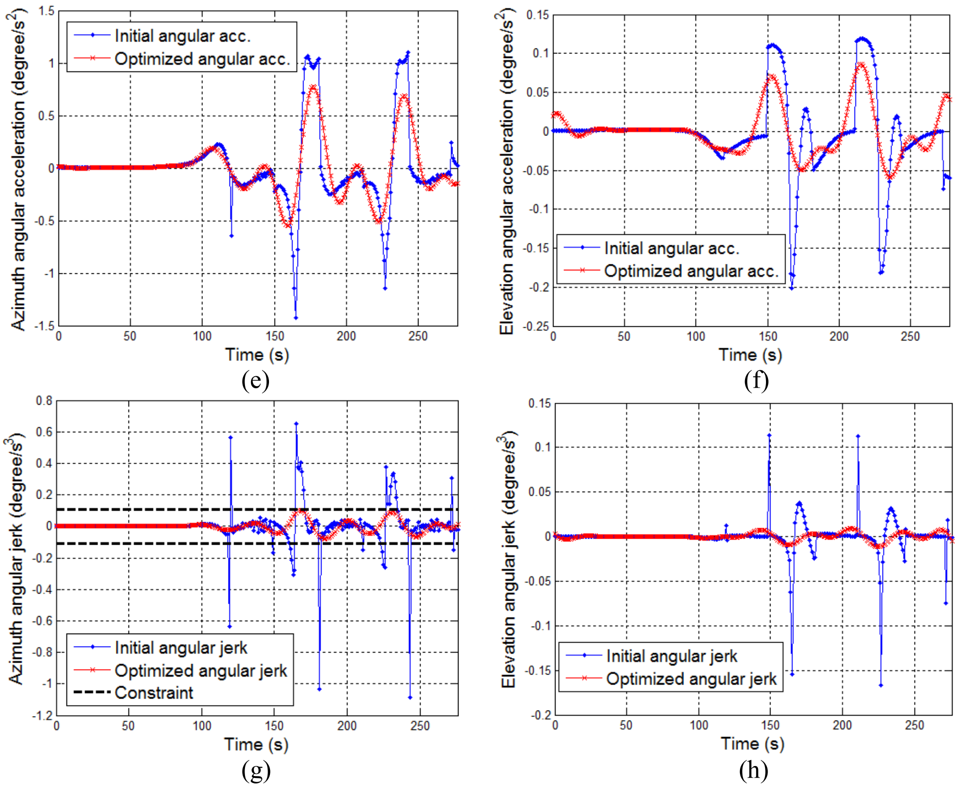


Figure 5. Initial and optimized results using set 2: (a) azimuth angle, (b) elevation angle, (c) azimuth angular velocity, (d) elevation angular velocity, (e) azimuth angular acceleration, (f) elevation angular acceleration, (g) azimuth angular jerk, and (h) elevation angular jerk.

B. Example 2: The Worst Case

The maximum angular velocity, acceleration, and jerk of the elevation profile, which are respectively 1.8753 degree/s, 1.2192 degree/s^2, and 0.6294 degree/s^3, are not critical. However, example 2 is one of the worst SAPs, because the maximum angular velocity and acceleration of the azimuth profile, which are respectively 29.8104 degree/s and 11.2012 degree/s^2, significantly violate the mechanical restrictions of 6 degree/s and 2 degree/s^2. The maximum azimuth angular jerk is 11.7871 degree/s^3, which is considered to affect the vibration. The number of angles is 512.

Tables 3 and 4 summarize the optimization results. The optimization of set 2 gives the best results in terms of the highest reduction ratio of the objective function and the least computation time. All constraints of optimizing the elevation profile are satisfied, but the azimuth angular velocity and acceleration violate the mechanical restrictions. The violations of the constraints are unsolvable in this example. In other words, the optimization results of the azimuth angular velocity and acceleration are maximum possible reductions within the off-pointing margin boundary.

Table 3. Optimization results of azimuth profile

Comparison criteria	Set 1	Set 2	Set 3
Reduction ratio of objective function (%)	92.2293	97.8009	95.2436
Maximum velocity (initial value: 29.8104) (degree/s)	17.6686	17.1891	17.9231
Maximum acceleration (initial value: 11.2012) (degree/s^2)	3.7231	2.6071	3.2808
Maximum jerk (initial value: 11.7871) (degree/s^3)	2.0669	1.0740	2.6247
Total time (s)	672	184	973

Table 4. Optimization results of elevation profile

Comparison criteria	Set 1	Set 2	Set 3
Reduction ratio of objective function (%)	78.4615	99.4740	99.5078
Maximum velocity (initial value: 1.8753) (degree/s)	1.9768	1.4428	1.4212
Maximum acceleration (initial value: 1.2192) (degree/s^2)	0.8865	0.1913	0.1862
Maximum jerk (initial value: 0.6294) (degree/s^3)	0.4575	0.0248	0.0238
Total time (s)	273	245	628

Fig. 6 shows the initial and optimized results of the azimuth angle, angular velocity, and angular acceleration in the violation section. In Fig. 6(a), the profile of the optimized angle makes the gentlest slope while satisfying the off-pointing margin boundary.

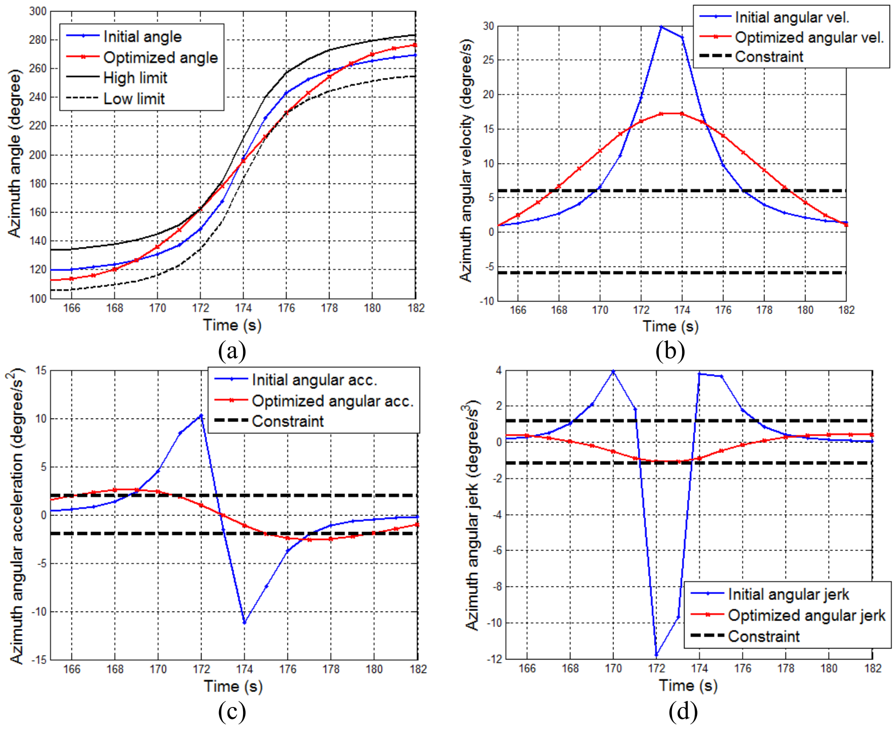


Figure 6. Violation section of the azimuth profile.

Fig. 7 shows the convergence histories of optimizing the azimuth and elevation profiles using set 2.

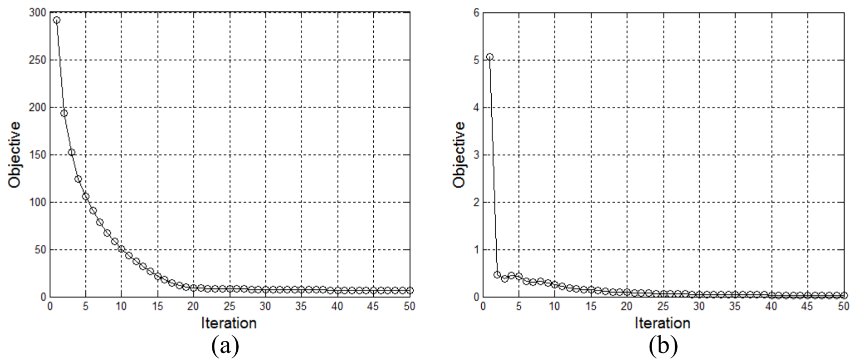


Figure 7. Optimization history of set 2: (a) azimuth angle, (b) elevation angle.

Fig. 8 shows the optimized results using set 2 compared to the initial values of the SAP.

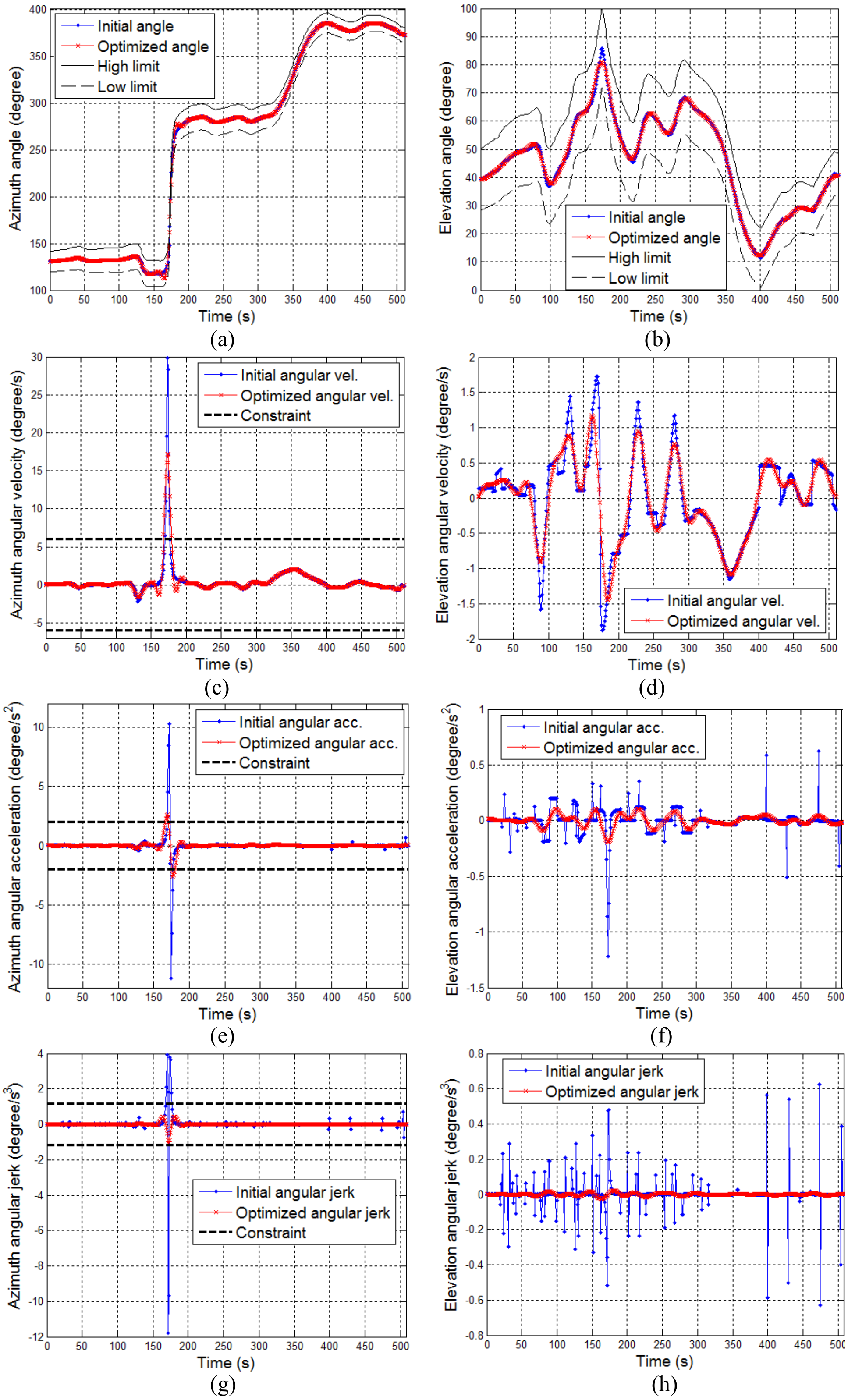


Figure 8. Initial and optimized results using set 2: (a) azimuth angle, (b) elevation angle, (c) azimuth angular velocity, (d) elevation angular velocity, (e) azimuth angular acceleration, (f) elevation angular acceleration, (g) azimuth angular jerk, and (h) elevation angular jerk.

V. Conclusion

This paper proposed the gradient-based optimization of the SAP to minimize the angular jerk. The sequential angle of the SAP is defined as the design variable, and the off-pointing margin boundary is used for the side limit. The angular velocity, acceleration, and jerk are determined from the sequential angle with respect to the sampling time. Three optimization problems were formulated by one objective function and three sets of constraints. The objective function was to minimize the total sum of the squared angular jerk. The first set of constraints was the angular velocity, and acceleration. The second set was the angular jerk. The third set was the angular velocity, acceleration, and jerk. Three optimization problems were compared in terms of optimization results obtained using two numerical examples. The second set of constraints is judged to be the best formulation of the optimization problems according to the comparison criteria of the computation time, the objective function, and the constraint satisfaction.

Acknowledgments

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